

磁化等离子体填充螺旋线的色散方程*

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利用等离子体流体理论和纵向场分量法, 结合螺旋线的导电面模型, 对在有限磁场作用下, 填充等离子体的螺旋线进行了严格的场分析. 在给出各区域电磁场分量表达式的基础上, 利用螺旋线的边界条件, 导出了磁化等离子体填充螺旋线中电磁波传播所满足的色散方程, 并对所导出的色散方程进行了讨论.

关键词: 有限磁场, 等离子体, 螺旋线, 色散方程

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1. 引 言

自上世纪 80 年代以来, 随着脉冲功率技术和相对论电子束技术的长足进展, 对等离子体填充高功率微波器件的研究已成为微波电子学领域内的前沿研究方向. 开始研究工作主要集中在圆柱波导、波纹波导和耦合腔等器件上^[1-9]. 近几年, 对于填充等离子体的螺旋线的研究, 也逐渐增多^[10-13]. 在已有的文献中, 为了简化分析, 通常都假定纵向引导磁场为无限大. 然而, 就器件的实际工作情况而言, 引导磁场总是有限的. 因此, 在有限磁场下, 对等离子体填充的螺旋线进行严格的场分析, 进而获得该类器件中电磁波传播的色散方程, 是一项必要的基础性研究工作. 文献 [11] 给出了轴对称模式下, 磁化等离子体填充螺旋线的色散方程, 并作了较为详细的数值计算和分析, 色散方程中的变量采用了矩阵的形式. 本文在考虑更加普遍的非对称模式情况下, 利用电磁场横向分量与纵向分量之间的关系, 建立了等离子体区和介质区的场分量表达式. 再利用螺旋导电面和波导壁的边界条件, 使各区域中的场匹配, 从而消去场幅值系数, 较简明地导出了磁化等离子体填充螺旋线的色散方程.

2. 磁化等离子体波导中电磁场的解^[6]

如图 1 所示, 在螺旋角为 ϕ 、半径为 r_w 的螺旋

线内充满了均匀的冷等离子体, 螺旋线外有一个半径为 r_w 的金属圆筒, 在圆筒与螺旋线间填满了各向同性介质, 其相对介电常数为 ϵ_d . 整个系统置于一轴向有限均匀磁场中.

在有限磁场下, 等离子体为一各向异性的介质. 由等离子体流体理论可知, 当忽略电子的碰撞时, 磁化等离子体的介电张量可表示为

$$[\epsilon] = \epsilon_0 \begin{bmatrix} \epsilon_1 & -\epsilon_2 & 0 \\ \epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}, \quad (1)$$

其中

$$\epsilon_1 = 1 - \frac{\xi^2}{1 - \tau^2}, \quad (2)$$

$$\epsilon_2 = \frac{j\tau\xi^2}{1 - \tau^2}, \quad (3)$$

$$\epsilon_3 = 1 - \xi^2, \quad (4)$$

$$\xi = \frac{\omega_p}{\omega}, \quad (5)$$

$$\tau = \frac{\omega_c}{\omega}. \quad (6)$$

$\omega_p = \sqrt{e^2 n_p / m_e \epsilon_0}$ 为等离子体频率, $\omega_c = \frac{eB_0}{m_e}$ 为电子回旋频率, j 为单位虚数, n_p 为等离子体密度, ϵ_0 为真空介电常数, ω 为波的角频率, e 和 m_e 分别为电子的电量和质量.

设波的传播因子为 $e^{j\omega t - \gamma z + jm\theta}$, 从 Maxwell 方程

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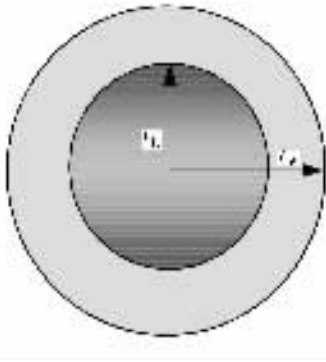


图 1 等离子体填充螺旋线的径向结构

可得到纵向场分量 E_z 和 H_z 所满足的波动方程

$$\nabla_{\perp}^2 E_z + aE_z = bH_z, \quad (7)$$

$$\nabla_{\perp}^2 H_z + cH_z = dE_z, \quad (8)$$

其中

$$a = (\gamma^2 + k^2 \epsilon_1) \epsilon_3 / \epsilon_1, \quad (9)$$

$$b = j\omega\mu_0 \gamma \epsilon_2 / \epsilon_1, \quad (10)$$

$$c = \gamma^2 + k^2 (\epsilon_1^2 + \epsilon_2^2) / \epsilon_1, \quad (11)$$

$$d = -j\omega\epsilon_0 \gamma \epsilon_2 \epsilon_3 / \epsilon_1, \quad (12)$$

$$k^2 = \omega^2 \epsilon_0 \mu_0, \quad (13)$$

$$\nabla_{\perp}^2 = \nabla^2 - \frac{\partial^2}{\partial z^2}. \quad (14)$$

从方程(7)(8)可知,在该波导中电磁波不能独立的分为 TE 波和 TM 波,而是互相耦合的混杂电磁波模式.

由方程(7)(8)可得

$$[\nabla_{\perp}^4 + (a+c)\nabla_{\perp}^2 + (ac-bd)]E_z = 0, \quad (15)$$

$$[\nabla_{\perp}^4 + (a+c)\nabla_{\perp}^2 + (ac-bd)]H_z = 0. \quad (16)$$

对(15)式分解,可得

$$(\nabla_{\perp}^2 + p_1^2)(\nabla_{\perp}^2 + p_2^2)E_z = 0. \quad (17)$$

设 $E_z = E_{z1} + E_{z2}$, 可得

$$(\nabla_{\perp}^2 + p_1^2)E_{z1} = 0, \quad (18)$$

$$(\nabla_{\perp}^2 + p_2^2)E_{z2} = 0. \quad (19)$$

在圆柱波导中,求解上述波动方程可得

$$E_z = A_1 J_m(p_1 r) + A_2 J_m(p_2 r), \quad (20)$$

$$H_z = A_1 h_1 J_m(p_1 r) + A_2 h_2 J_m(p_2 r), \quad (21)$$

其中

$$h_{1,2} = \frac{(\gamma^2 + k^2 \epsilon_1) \frac{\epsilon_3}{\epsilon_1} - p_{1,2}^2}{j\omega\mu_0 \gamma \frac{\epsilon_2}{\epsilon_1}}, \quad (22)$$

$$p_{1,2}^2 = \frac{1}{2\epsilon_1} [\gamma^2 (\epsilon_1 + \epsilon_3) + k^2 (\epsilon_1 \epsilon_3 + \epsilon_1^2 + \epsilon_2^2)]$$

$$\pm \frac{1}{2\epsilon_1} \left\{ \left[\gamma^2 (\epsilon_3 - \epsilon_1) + k^2 (\epsilon_1 \epsilon_3 - \epsilon_1^2 - \epsilon_2^2) \right] + 4k^2 \gamma^2 \epsilon_2^2 \epsilon_3 \right\}^{1/2}. \quad (23)$$

利用 E_z 和 H_z , 通过下面电磁场横向分量与纵向分量之间的关系可求得横向场分量

$$E_r = \frac{1}{D} \left(-\gamma K^2 \frac{\partial E_z}{\partial r} + j\gamma k_g^2 \frac{1}{r} \frac{\partial E_z}{\partial \theta} + \omega\mu_0 k_g^2 \frac{\partial H_z}{\partial r} - j\omega\mu_0 K^2 \frac{1}{r} \frac{\partial H_z}{\partial \theta} \right), \quad (24)$$

$$E_{\theta} = \frac{1}{D} \left(-j\gamma k_g^2 \frac{\partial E_z}{\partial r} - \gamma K^2 \frac{1}{r} \frac{\partial E_z}{\partial \theta} + j\omega\mu_0 K^2 \frac{\partial H_z}{\partial r} + \omega\mu_0 k_g^2 \frac{1}{r} \frac{\partial H_z}{\partial \theta} \right), \quad (25)$$

$$H_r = \frac{1}{D} \left(\omega\epsilon_0 \epsilon_g \gamma^2 \frac{\partial E_z}{\partial r} + j\omega\epsilon_0 (\epsilon_1 K^2 - \epsilon_g k_g^2) \frac{1}{r} \frac{\partial E_z}{\partial \theta} - \gamma K^2 \frac{\partial H_z}{\partial r} + j\gamma k_g^2 \frac{1}{r} \frac{\partial H_z}{\partial \theta} \right), \quad (26)$$

$$H_{\theta} = \frac{1}{D} \left(-j\omega\epsilon_0 (\epsilon_1 K^2 - \epsilon_g k_g^2) \frac{\partial E_z}{\partial r} + \omega\epsilon_0 \epsilon_g \gamma^2 \frac{1}{r} \frac{\partial E_z}{\partial \theta} - j\gamma k_g^2 \frac{\partial H_z}{\partial r} - \gamma K^2 \frac{1}{r} \frac{\partial H_z}{\partial \theta} \right), \quad (27)$$

其中

$$D = K^4 - k_g^4, \quad (28)$$

$$K^2 = \gamma^2 + k^2 \epsilon_1, \quad (29)$$

$$k_g^2 = k^2 \epsilon_g, \quad (30)$$

$$\epsilon_g = j\epsilon_2. \quad (31)$$

3. 色散方程

对图 1 所示的结构,利用纵向场分量法,可求得系统各区域中电磁场分量的表达式(以下各式中均省去了因子 $e^{j\omega t - \gamma z + jm\theta}$).

在等离子体区($r \leq r_h$):

$$E_z = A_1 J_m(p_1 r) + A_2 J_m(p_2 r), \quad (32)$$

$$E_r = \frac{1}{D} \left\{ -\gamma K^2 [A_1 p_1 J_m'(p_1 r) + A_2 p_2 J_m'(p_2 r)] - \gamma k_g^2 \frac{m}{r} [A_1 J_m(p_1 r) + A_2 J_m(p_2 r)] + \omega\mu_0 k_g^2 [A_1 h_1 p_1 J_m'(p_1 r) + A_2 h_2 p_2 J_m'(p_2 r)] + \omega\mu_0 K^2 \frac{m}{r} [A_1 h_1 J_m(p_1 r) + A_2 h_2 J_m(p_2 r)] \right\}, \quad (33)$$

$$E_{\theta} = \frac{1}{D} \left\{ -j\gamma k_g^2 [A_1 p_1 J'_m(p_1 r) + A_2 p_2 J'_m(p_2 r)] - j\gamma K^2 \frac{m}{r} [A_1 J_m(p_1 r) + A_2 J_m(p_2 r)] \right. \\ \left. + j\omega\mu_0 K^2 [A_1 h_1 p_1 J'_m(p_1 r) + A_2 h_2 p_2 J'_m(p_2 r)] + j\omega\mu_0 k_g^2 \frac{m}{r} [A_1 h_1 J_m(p_1 r) + A_2 h_2 J_m(p_2 r)] \right\}, \quad (34)$$

$$H_z = A_1 h_1 J_m(p_1 r) + A_2 h_2 J_m(p_2 r), \quad (35)$$

$$H_r = \frac{1}{D} \left\{ \omega\epsilon_0 \epsilon_g \gamma^2 [A_1 p_1 J'_m(p_1 r) + A_2 p_2 J'_m(p_2 r)] - \omega\epsilon_0 (\epsilon_1 K^2 - \epsilon_g k_g^2) \frac{m}{r} [A_1 J_m(p_1 r) + A_2 J_m(p_2 r)] \right. \\ \left. - \gamma K^2 [A_1 h_1 p_1 J'_m(p_1 r) + A_2 h_2 p_2 J'_m(p_2 r)] - \gamma k_g^2 \frac{m}{r} [A_1 h_1 J_m(p_1 r) + A_2 h_2 J_m(p_2 r)] \right\}, \quad (36)$$

$$H_{\theta} = \frac{1}{D} \left\{ -j\omega\epsilon_0 (\epsilon_1 K^2 - \epsilon_g k_g^2) [A_1 p_1 J'_m(p_1 r) + A_2 p_2 J'_m(p_2 r)] + j\omega\epsilon_0 \epsilon_g \gamma^2 \frac{m}{r} [A_1 J_m(p_1 r) + A_2 J_m(p_2 r)] \right. \\ \left. - j\gamma k_g^2 [A_1 h_1 p_1 J'_m(p_1 r) + A_2 h_2 p_2 J'_m(p_2 r)] - j\gamma K^2 \frac{m}{r} [A_1 h_1 J_m(p_1 r) + A_2 h_2 J_m(p_2 r)] \right\}. \quad (37)$$

在各向同性介质区 ($r_h \leq r \leq r_w$):

$$E_z = A_3 J_m(pr) + A_4 N_m(pr), \quad (38)$$

$$E_r = \frac{1}{p^2} \left\{ -\gamma p [A_3 J'_m(pr) + A_4 N'_m(pr)] + \omega\mu_0 \frac{m}{r} [A_5 J_m(pr) + A_6 N_m(pr)] \right\}, \quad (39)$$

$$E_{\theta} = \frac{1}{p^2} \left\{ -j\gamma \frac{m}{r} [A_3 J_m(pr) + A_4 N_m(pr)] + j\omega\mu_0 p [A_5 J'_m(pr) + A_6 N'_m(pr)] \right\}, \quad (40)$$

$$H_z = A_5 J_m(pr) + A_6 N_m(pr), \quad (41)$$

$$H_r = \frac{1}{p^2} \left\{ -\omega\epsilon_0 \epsilon_d \frac{m}{r} [A_3 J_m(pr) + A_4 N_m(pr)] - \gamma p [A_5 J'_m(pr) + A_6 N'_m(pr)] \right\}, \quad (42)$$

$$H_{\theta} = \frac{1}{p^2} \left\{ -j\omega\epsilon_0 \epsilon_d p [A_3 J'_m(pr) + A_4 N'_m(pr)] - j\gamma \frac{m}{r} [A_5 J_m(pr) + A_6 N_m(pr)] \right\}, \quad (43)$$

其中

$$p^2 = \gamma^2 + k^2 \epsilon_d. \quad (44)$$

将上述两区域中场分量的表达式, 代入以下各边界条件:

$$E_z(r = r_h^-) = E_z(r = r_h^+), \quad (45)$$

$$E_{\theta}(r = r_h^-) = E_{\theta}(r = r_h^+), \quad (46)$$

$$E_z(r = r_h^-) + E_{\theta}(r = r_h^-) \text{ctg}\phi \\ = E_z(r = r_h^+) + E_{\theta}(r = r_h^+) \text{ctg}\phi = 0, \quad (47)$$

$$H_z(r = r_h^-) + H_{\theta}(r = r_h^-) \text{ctg}\phi \\ = H_z(r = r_h^+) + H_{\theta}(r = r_h^+) \text{ctg}\phi, \quad (48)$$

$$E_z(r = r_w) = 0, \quad (49)$$

$$E_{\theta}(r = r_w) = 0, \quad (50)$$

可分别得到各场分量的相对幅值(详见附录):

$$A_1 = \frac{f_2 + DJ_m(p_2 r_h) \text{tg}\phi}{J_m(p_1 r_h) f_2 - J_m(p_2 r_h) f_1} RA_3, \quad (51)$$

$$A_2 = \frac{f_1 + DJ_m(p_1 r_h) \text{tg}\phi}{J_m(p_1 r_h) f_2 - J_m(p_2 r_h) f_1} RA_3, \quad (52)$$

$$A_4 = -\frac{J_m(pr_w)}{N_m(pr_w)} A_3, \quad (53)$$

$$A_5 = jT \frac{\left(\beta \frac{m}{r_h} + p^2 \text{tg}\phi \right)}{\omega\mu_0 p} \cdot \frac{N'_m(pr_w)}{N_m(pr_w)} A_3, \quad (54)$$

$$A_6 = -jT \frac{\left(\beta \frac{m}{r_h} + p^2 \text{tg}\phi \right)}{\omega\mu_0 p} \cdot \frac{J'_m(pr_w)}{N_m(pr_w)} A_3. \quad (55)$$

将各场分量相对幅值系数代入螺旋边界条件(48)式, 消去 A_3 , 可求得色散方程为

$$\frac{1}{p^2 D} \left[C_0 + C_1 \left(\frac{m}{r_h} \right) + C_2 \left(\frac{m}{r_h} \right)^2 + C_3 \left(\frac{m}{r_h} \right)^3 \right] = 0, \quad (56)$$

其中,

$$C_0 = [\beta k_g^2 (p^2 Q \text{tg}\phi - k^2 \epsilon_d F \text{ctg}\phi) - pK^2 \omega^2 \mu_0^2 h_1 h_2 \\ - pk_g^2 (\epsilon_1 K^2 - \epsilon_g k_g^2)] \cdot [p_2 J_m(p_1 r_h) J'_m(p_2 r_h) \\ - p_1 J_m(p_2 r_h) J'_m(p_1 r_h)] + j\omega\mu_0 [K^2 (p^2 Q \text{tg}\phi \\ - k^2 \epsilon_d F \text{ctg}\phi) - p\beta k_g^2] \\ \cdot [h_2 p_2 J_m(p_1 r_h) J'_m(p_2 r_h) \\ - h_1 p_1 J_m(p_2 r_h) J'_m(p_1 r_h)] \\ + j\omega\mu_0 \beta k_g^2 p^2 [h_1 p_2 J_m(p_1 r_h) J'_m(p_2 r_h) \\ - h_2 p_1 J_m(p_2 r_h) J'_m(p_1 r_h)]$$

$$+ j\omega\mu_0(h_1 - h_2) \int p D \text{tg}\phi J_m(p_1 r_h) J_m(p_2 r_h) - k^2 \epsilon_1 p_1 p_2 p \text{ctg}\phi J'_m(p_1 r_h) J'_m(p_2 r_h) \int], \quad (57)$$

$$C_1 = 2Q\beta^2 k_g^2 \int p_2 J_m(p_1 r_h) J'_m(p_2 r_h) - p_1 J_m(p_2 r_h) J'_m(p_1 r_h) \int + 2j\omega\mu_0 Q K^2 \int h_2 p_2 J_m(p_1 r_h) J'_m(p_2 r_h) - h_1 p_1 J_m(p_2 r_h) J'_m(p_1 r_h) + j\omega\mu_0(h_2 - h_1) J_m(p_1 r_h) J_m(p_2 r_h) \int (p^2 Q \text{tg}\phi - k^2 \epsilon_d F \text{ctg}\phi) k_g^2 - 2\beta K^2 p \int - p\beta k^2 \epsilon_1 \text{ctg}\phi \int p_2 J_m(p_1 r_h) J'_m(p_2 r_h) - p_1 J_m(p_2 r_h) J'_m(p_1 r_h) \int - j\omega\mu_0 p k_g^2 \text{ctg}\phi \int h_1 p_2 J_m(p_1 r_h) J'_m(p_2 r_h) - h_2 p_1 J_m(p_2 r_h) J'_m(p_1 r_h) \int - \omega^2 \mu_0^2 p \beta \text{ctg}\phi h_1 h_2 \int p_2 J_m(p_1 r_h) J'_m(p_2 r_h) - p_1 J_m(p_2 r_h) J'_m(p_1 r_h) \int], \quad (58)$$

$$C_2 = j\omega\mu_0(h_2 - h_1) \int 2Q\beta k_g^2 - p\beta^2 \text{ctg}\phi \int J_m(p_1 r_h) \times J_m(p_2 r_h) + \frac{Q}{p} \beta^2 \text{ctg}\phi \int \beta k_g^2 \int p_2 J_m(p_1 r_h) \times J'_m(p_2 r_h) - p_1 J_m(p_2 r_h) J'_m(p_1 r_h) \int + j\omega\mu_0 K^2 \int h_2 p_2 J_m(p_1 r_h) J'_m(p_2 r_h) - h_1 p_1 J_m(p_2 r_h) J'_m(p_1 r_h) \int], \quad (59)$$

$$C_3 = j\omega\mu_0(h_2 - h_1) \frac{Q}{p} \beta^2 k_g^2 \text{ctg}\phi J_m(p_1 r_h) J_m(p_2 r_h), \quad (60)$$

$$Q = \frac{J_m(pr_h) N'_m(pr_w) - J'_m(pr_w) N_m(pr_h)}{J'_m(pr_h) N'_m(pr_w) - J'_m(pr_w) N'_m(pr_h)}, \quad (61)$$

$$F = \frac{J'_m(pr_h) N_m(pr_w) - J_m(pr_w) N'_m(pr_h)}{J_m(pr_h) N_m(pr_w) - J_m(pr_w) N'_m(pr_h)}. \quad (62)$$

方程(56)式就是充磁化等离子体螺旋线中电磁波传播应满足的色散方程的普遍形式,基于这一普遍形式,可以得到各种特定条件下的色散方程.

1) 当 $n_p = 0$, 即未填充等离子体时, 有 $\epsilon_1 = \epsilon_3 = 1$, $\epsilon_2 = 0$, 对称模式 ($m = 0$) 的色散方程可简化为

$$p_1^2 \text{tg}^2 \phi \frac{J_0(p_1 r_h)}{J'_0(p_1 r_h)} \left[\frac{p}{p_1} \cdot \frac{J'_0(p_1 r_h)}{J_0(p_1 r_h)} Q - 1 \right] = k^2 \epsilon_1 \left[\frac{p_1}{p} \cdot \frac{\epsilon_d}{\epsilon_1} F - \frac{J'_0(p_1 r_h)}{J_0(p_1 r_h)} \right], \quad (63)$$

可以证明(63)式与文献[14]中的结果一致.

2) 当 $n_p \neq 0$, $B_0 = 0$ 时, 即未磁化等离子体的情况, 此时等离子体为一各向同性的色散媒质, $\epsilon_1 = \epsilon_3 = 1 - \xi^2$, $\epsilon_2 = 0$. 对称模式的色散方程与(63)式相同.

3) 当 $n_p \neq 0$, $B_0 \rightarrow \infty$, 此时等离子体为一各向异

性的媒质, $\epsilon_1 = 1$, $\epsilon_2 = 0$, $\epsilon_3 = 1 - \xi^2$, $h_1 = 0$, $h_2 \rightarrow \infty$, 将上述结果代入(56)式并取极限得

$$\frac{1}{p^2 D} \left\{ \left[(p^2 Q \text{tg}\phi - k^2 \epsilon_d F \text{ctg}\phi) p_2 K^2 J_m(p_1 r_h) \times J'_m(p_2 r_h) - p D J_m(p_1 r_h) J_m(p_2 r_h) \text{tg}\phi + p_1 p_2 p k^2 \epsilon_1 \text{ctg}\phi J'_m(p_1 r_h) J'_m(p_2 r_h) \right] + \frac{m}{r_h} \left[2 Q K^2 \beta p_2 J_m(p_1 r_h) J'_m(p_2 r_h) - 2 \beta K^2 p J_m(p_1 r_h) J_m(p_2 r_h) \right] + \left[\frac{Q}{p} \beta^2 K^2 p_2 J_m(p_1 r_h) J'_m(p_2 r_h) - p \beta^2 J_m(p_1 r_h) J_m(p_2 r_h) \right] \left(\frac{m}{r_h} \right)^2 \text{ctg}\phi \right\} = 0, \quad (64)$$

(64)式在对称模式 ($m = 0$) 下与文献[13]结果一致.

4) 当 $n_p \neq 0$, B_0 为有限时, 在对称模式 ($m = 0$) 下, 色散方程(56)式可简化为 $C_0 = 0$.

4. 结 论

本文在有限磁场下, 对填充等离子体的螺旋线进行了严格的场分析, 利用螺旋导电面的边界条件导出了磁化等离子体填充螺旋线中电磁波传播所满足的普遍情况下的色散方程, 它是进一步进行数值计算和理论分析的基础. 通过对四种常见的特殊情况(无等离子体、磁场为零、磁场为无穷大、对称模式)的分析与讨论, 得到了上述几种特殊情况下简化的色散方程. 研究表明, 简化所得到的结果与已有的研究文献一致.

附 录

将场分量表达式代入边界条件, 可求得各场分量相对幅值系数之间的关系. 将(38)式代入(49)式, 可得

$$E_z(r = r_w) = A_3 J_m(pr_w) + A_4 N_m(pr_w) = 0, \quad (A1)$$

因此

$$A_4 = - \frac{J_m(pr_w)}{N_m(pr_w)} A_3. \quad (A2)$$

将(40)式代入(50)式, 可得

$$E_\theta(r = r_w) = \frac{1}{p^2} \left\{ -j\gamma \frac{m}{r} \left[A_3 J_m(pr_w) + A_4 N_m(pr_w) \right] + j\omega\mu_0 p \left[A_3 J'_m(pr_w) + A_4 N'_m(pr_w) \right] \right\} = 0. \quad (A3)$$

利用(A1)式(A3)式可化为

$$A_6 = -\frac{J'_m(pr_w)}{N'_m(pr_w)}A_5. \quad (A4)$$

将(38)式和(40)式代入(47)式,可得

$$A_3 J_m(pr_h) + A_4 N_m(pr_h) + \frac{ctg\phi}{p^2} \left\{ -j\gamma \frac{m}{r_h} [A_3 J_m(pr_h) + A_4 N_m(pr_h)] + j\omega\mu_0 p [A_5 J'_m(pr_h) + A_6 N'_m(pr_h)] \right\} = 0. \quad (A5)$$

将(A2)(A4)两式代入(A5)式,消去 A_4 和 A_6 ,可得

$$A_5 = jT \frac{\left(\beta \frac{m}{r_h} + p^2 tg\phi\right)}{\omega\mu_0 p} \cdot \frac{N'_m(pr_w)}{N_m(pr_w)} A_3, \quad (A6)$$

其中

$$\beta = \frac{\gamma}{j}, \quad (A7)$$

$$T = \frac{J_m(pr_h)N_m(pr_w) - J_m(pr_w)N_m(pr_h)}{J'_m(pr_h)N'_m(pr_w) - J'_m(pr_w)N'_m(pr_h)}. \quad (A8)$$

将(A6)式代入(A4)式,可得

$$A_6 = -jT \frac{\left(\beta \frac{m}{r_h} + p^2 tg\phi\right)}{\omega\mu_0 p} \cdot \frac{J'_m(pr_w)}{N_m(pr_w)} A_3. \quad (A9)$$

将(32)(34)两式代入(47)式,可得

$$A_1 J_m(p_1 r_h) + A_2 J_m(p_2 r_h) + \frac{ctg\phi}{D} \left\{ -j\gamma k_g^2 [A_1 p_1 J'_m(p_1 r_h) + A_2 p_2 J'_m(p_2 r_h)] - j\gamma K^2 \frac{m}{r_h} [A_1 J_m(p_1 r_h) + A_2 J_m(p_2 r_h)] + j\omega\mu_0 K^2 [A_1 h_1 p_1 J'_m(p_1 r_h) + A_2 h_2 p_2 J'_m(p_2 r_h)] \right\} = 0. \quad (A10)$$

$$+ j\omega\mu_0 k_g^2 \frac{m}{r} [A_1 h_1 J_m(p_1 r_h) + A_2 h_2 J_m(p_2 r_h)] \left. \right\} = 0. \quad (A10)$$

由(A10)式可求得

$$A_2 = \frac{f_2 + DJ_m(p_2 r_h) \lambda g\phi}{f_1 + DJ_m(p_1 r_h) \lambda g\phi} A_1, \quad (A11)$$

其中

$$f_1 = \beta k_g^2 p_1 J'_m(p_1 r_h) + \beta K^2 \frac{m}{r_h} J_m(p_1 r_h) + jK^2 \omega\mu_0 h_1 p_1 J'_m(p_1 r_h) + j \frac{m}{r_h} k_g^2 \omega\mu_0 h_1 J_m(p_1 r_h), \quad (A12)$$

$$f_2 = \beta k_g^2 p_2 J'_m(p_2 r_h) + \beta K^2 \frac{m}{r_h} J_m(p_2 r_h) + jK^2 \omega\mu_0 h_2 p_2 J'_m(p_2 r_h) + j \frac{m}{r_h} k_g^2 \omega\mu_0 h_2 J_m(p_2 r_h). \quad (A13)$$

将(32)(38)两式代入(45)式,可得

$$A_1 J_m(p_1 r_h) + A_2 J_m(p_2 r_h) = A_3 J_m(pr_h) + A_4 N_m(pr_h). \quad (A14)$$

将(A2)(A11)两式代入(A14)式,消去 A_2 和 A_4 ,可得

$$A_1 = \frac{f_2 + DJ_m(p_2 r_h) \lambda g\phi}{J_m(p_1 r_h) f_2 - J_m(p_2 r_h) f_1} RA_3, \quad (A15)$$

其中

$$R = \frac{J_m(pr_h)N_m(pr_w) - J_m(pr_w)N_m(pr_h)}{N_m(pr_w)}. \quad (A16)$$

将(A11)式代入(A15)式,可得

$$A_2 = \frac{f_1 + DJ_m(p_1 r_h) \lambda g\phi}{J_m(p_1 r_h) f_2 - J_m(p_2 r_h) f_1} RA_3. \quad (A17)$$

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Dispersion equation of a helical slow wave structure filled with magnetized plasma *

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Abstract

Analytical theory of electromagnetic field in a helical slow wave structure filled with plasma immersed in a finite axial magnetic field is presented in this paper. By means of helix sheath model and the boundary conditions, the dispersion equation of a magnetized plasma loaded helical slow wave structure is derived. Three particular cases (without plasma, without axial magnetic field and in infinite axial magnetic field) are discussed and the corresponding dispersion equations are deduced. It is found that the simplified results are in accord and with published references.

Keywords : finite magnetic field, plasma, helix, dispersion equation

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