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准坐标下广义非保守系统 Lagrange 方程的积分因子与守恒定理*

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用积分因子方法研究准坐标下广义非保守系统 Lagrange 方程的守恒定理.列写系统的运动微分方程,给出它的积分因子的定义.研究守恒量存在的必要条件.建立系统的守恒定理及其逆定理,并举例说明结果的应用.

关键词:准坐标,Lagrange 方程,积分因子,守恒量

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1. 引 言

动力学系统的对称性,亦称不变性.用对称性研究守恒量是数学物理的一种近代方法.主要有 Noethor 理论[1],Lie 对称性[2],Mei 方法[3]和 Hojman 方法[4].近年来人们将这些方法推广到广义经典力学,已取得一系列重要成果[5—16].

1984 年,Djukic′提出了构造非保守动力学系统守恒量的积分因子方法[17],即将运动微分方程乘以适当的积分因子来求系统的守恒量.这种方法与上述几种著名方法相比,限制条件少、容易运算,有广泛应用价值,在分析力学的积分理论中非常有发展前途.该方法在研究各类运动方程的守恒量方面已有进展[18—22].

本文将积分因子方法推广到广义经典力学,寻求准坐标下广义 Lagrange 方程的守恒量.

2. 系统的运动微分方程

假设广义力学系统的位形由 n 个广义坐标 q_1,q_2,…,q_n 确定.选准速度为广义速度的线性式,即

$$\omega_s = \sum_{r=1}^{n} a_{sr}(\boldsymbol{q})\dot{q}_r, \tag{1}$$

设由(1)式可解出广义速度

$$\dot{q}_s = \sum_{r=1}^{n} b_{sr}(\boldsymbol{q})\omega_r, \tag{2}$$

则系统的运动微分方程可表示为[12]

$$\frac{\partial \tilde{L}}{\partial \pi_s} + \sum_{\alpha=1}^{\delta}(-1)^{\alpha}\frac{\mathrm{d}^{\alpha}}{\mathrm{d}t^{\alpha}}\frac{\partial \tilde{L}}{\partial \overset{(\alpha-1)}{\omega_s}} = \tilde{R}_s$$

$$(s = 1,2,\cdots,n). \tag{3}$$

其中 $\tilde{L}$ 为用准速度表示的 Lagrange 函数,它与广义坐标下的 Lagrange 函数 L 有如下关系:

$$\tilde{L}(t,q_s,\omega_s,\dot{\omega}_s,\ddot{\omega}_s,\cdots,\overset{(\delta-1)}{\omega_s})$$

$$= L\Big[t,q_s,\Big(\sum_{r=1}^{n} b_{sr}\omega_r\Big),\Big(\sum_{r=1}^{n} b_{sr}\omega_r\Big)',\cdots,\Big(\sum_{r=1}^{n} b_{sr}\omega_r\Big)^{(\delta-1)}\Big], \tag{4}$$

对准坐标 π_s 的偏导数为

$$\frac{\partial}{\partial \pi_s} = \sum_{r=1}^{n} b_{sr}\frac{\partial}{\partial q_r}, \tag{5}$$

而 $\tilde{Q}_s$ 为用准速度表示的非势广义力,它与非势广义力 Q_s 有如下关系:

$$\tilde{Q}_s = \sum_{k=1}^{n} Q_s b_{ks}, \tag{6}$$

$$\tilde{R}_s = -\tilde{Q}_s + \sum_{l=1}^{n}\sum_{k=1}^{n}(-1)^{\alpha}\sum_{\alpha=0}^{\delta-1}\gamma_{ks}^{l}\frac{\mathrm{d}^{\alpha}}{\mathrm{d}t^{\alpha}}\frac{\partial \tilde{L}}{\partial \overset{(\alpha)}{\omega_l}}\omega_k. \tag{7}$$

Boltzmann 三标记符号

$$\gamma_{ks}^{l} = \sum_{m=1}^{n}\sum_{r=1}^{n}\Big(\frac{\partial a_{lm}}{\partial q_r} - \frac{\partial a_{lr}}{\partial q_m}\Big)b_{rk}b_{ms}, \tag{8}$$

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且

$$\gamma_{ks}^{l} = -\gamma_{sk}^{l}. \tag{9}$$

定义准坐标下广义力学系统的广义动量为

$$\tilde{p}_{s/\alpha} = \sum_{j=0}^{\delta-\alpha}(-1)^j \frac{\mathrm{d}^j}{\mathrm{d}t^j}\left(\frac{\partial \tilde{L}}{\partial \overset{(j+\alpha-1)}{\omega_s}}\right),$$

$$(\alpha = 1,2,\cdots,\delta). \tag{10}$$

3. 积分因子

如果不变式

$$\sum_{s=1}^{n}\left[\frac{\partial \tilde{L}}{\partial \pi_s} + \sum_{\alpha=1}^{\delta}(-1)^\alpha \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha}\left(\frac{\partial \tilde{L}}{\partial \overset{(\alpha-1)}{\omega_s}}\right) - \tilde{R}_s\right]\xi_s$$

恒等地变为

$$\sum_{s=1}^{n}\left[\frac{\partial \tilde{L}}{\partial \pi_s} + \sum_{\alpha=1}^{\delta}(-1)^\alpha \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha}\left(\frac{\partial \tilde{L}}{\partial \overset{(\alpha-1)}{\omega_s}}\right) - \tilde{R}_s\right]\xi_s$$

$$\equiv \frac{\mathrm{d}}{\mathrm{d}t}\left[\tilde{L}\tau + \sum_{s=1}^{n}\sum_{\alpha=1}^{\delta}\tilde{p}_{s/\alpha}\frac{\mathrm{d}^{\alpha-1}}{\mathrm{d}t^{\alpha-1}}(\xi_s - \omega_s\tau) + \lambda\right]$$

$$+ \sum_{s=1}^{n}\mu_s\left[\frac{\partial \tilde{L}}{\partial \pi_s} + \sum_{\alpha=1}^{\delta}(-1)^\alpha \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha}\left(\frac{\partial \tilde{L}}{\partial \overset{(\alpha-1)}{\omega_s}}\right) - \tilde{R}_s\right]. \tag{11}$$

其中 τ,λ 和 μ_s 为 $t,\boldsymbol{q},\omega,\dot{\omega},\cdots,\overset{(\delta-1)}{\omega}$ 的函数,则称 $\xi_s = \xi_s(t,\boldsymbol{q},\omega,\dot{\omega},\cdots,\overset{(\delta-1)}{\omega})$ 为准坐标下广义非保守系统(3)的积分因子.

4. 守恒量存在定理

联合(3)式和(11)式,有

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\tilde{L}\tau + \sum_{s=1}^{n}\sum_{\alpha=1}^{\delta}\tilde{p}_{s/\alpha}\frac{\mathrm{d}^{\alpha-1}}{\mathrm{d}t^{\alpha-1}}(\xi_s - \omega_s\tau) + \lambda\right]$$

$$= \sum_{s=1}^{n} -\mu_s\left[\frac{\partial \tilde{L}}{\partial \pi_s} + \sum_{\alpha=1}^{\delta}(-1)^\alpha \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha}\left(\frac{\partial \tilde{L}}{\partial \overset{(\alpha-1)}{\omega_s}}\right) - \tilde{R}_s\right]. \tag{12}$$

于是有下面的定理.

定理 1 如果函数 ξ_s 是 Lagrange 方程(3)的积分因子,则准坐标下广义非保守系统(3)存在守恒量(初积分)

$$I = \tilde{L}\tau + \sum_{s=1}^{n}\sum_{\alpha=1}^{\delta}\tilde{p}_{s/\alpha}\frac{\mathrm{d}^{\alpha-1}}{\mathrm{d}t^{\alpha-1}}(\xi_s - \omega_s\tau) + \lambda$$

$$= \mathrm{const}. \tag{13}$$

证明 将上式对时间 t 求导数,并考虑方程(3)和(12),得

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\tilde{L}\tau + \sum_{s=1}^{n}\sum_{\alpha=1}^{\delta}\tilde{p}_{s/\alpha}\frac{\mathrm{d}^{\alpha-1}}{\mathrm{d}t^{\alpha-1}}(\xi_s - \omega_s\tau) + \lambda\right]$$

$$= \sum_{s=1}^{n} -\mu_s\left[\frac{\partial \tilde{L}}{\partial \pi_s} + \sum_{\alpha=1}^{\delta}(-1)^\alpha \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha}\left(\frac{\partial \tilde{L}}{\partial \overset{(\alpha-1)}{\omega_s}}\right) - \tilde{R}_s\right]$$

$$= 0. \tag{14}$$

对于已知的动力学系统(3),如果 ξ_s 是方程(3)的积分因子,则每一组函数 ξ_s,τ,λ 和 μ_s 一定满足必要条件(12).利用方程(3),条件(12)可写为

$$\frac{\partial \tilde{L}}{\partial t}\tau + \sum_{s=1}^{n}\frac{\partial \tilde{L}}{\partial \pi_s}\xi_s + \sum_{s=1}^{n}\sum_{i=1}^{\delta}\frac{\partial \tilde{L}}{\partial \overset{(i-1)}{\omega_s}}\eta^{(i)}$$

$$+ \tilde{L}\dot{\tau} - \sum_{s=1}^{n}\tilde{R}_s(\xi_s - \omega_s\tau) + \dot{\lambda} + \tilde{\Phi} = 0. \tag{15}$$

其中

$$\eta^{(1)} = \dot{\xi}_s - \omega_s\dot{\tau},$$

$$\eta^{(2)} = \frac{\mathrm{d}}{\mathrm{d}t}\eta^{(1)} - \dot{\omega}_s\dot{\tau},$$

$$\cdots,$$

$$\eta^{(\delta)} = \frac{\mathrm{d}}{\mathrm{d}t}\eta^{(\delta-1)} - \overset{(\delta-1)}{\omega_s}\dot{\tau}, \tag{16}$$

$$\tilde{\Phi} = \sum_{s=1}^{n} -\mu_s\left[\frac{\partial \tilde{L}}{\partial \pi_s} + \sum_{\alpha=1}^{\delta}(-1)^\alpha \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha}\left(\frac{\partial \tilde{L}}{\partial \overset{(\alpha-1)}{\omega_s}}\right) - \tilde{R}_s\right]. \tag{17}$$

显然,如果函数组 ξ,τ,λ 和 μ_s 满足必要条件(15),并使(13)式的等号右边成为常数,则有如下结果.

推论 对于满足必要条件(15)的每个非奇异函数组 ξ_s,τ,λ 和 μ_s 存在已知准坐标下广义非保守系统(3)的守恒量(13).

通过方程(15)的积分可以给出一个守恒量的非奇异函数组 ξ_s,τ,λ 和 μ_s.在任何情况下,如果得到方程(15)的任意解,其中不包含任何积分常数,称此解为方程(15)的一个函数解,由推论可以得到广义非保守系统(3)的一个守恒量(初积分).

5. 逆定理

对于准坐标下广义 Lagrange 系统,如果已知系统的第一积分,便可由积分因子逆定理找到函数 λ, τ 和积分因子 ξ_s.

假设已知广义非保守系统(3)有初积分

$$I = I(t,q_s,\omega_s,\dot{\omega}_s,\ddot{\omega}_s,\cdots,\overset{(2\delta-2)}{\omega_s}) = \mathrm{const}, \tag{18}$$

于是,有

$$\frac{\mathrm{d}I}{\mathrm{d}t}=\frac{\partial I}{\partial t}+\sum_{s=1}^{n}\frac{\partial I}{\partial \pi_s}\omega_s+\sum_{s=1}^{n}\frac{\partial I}{\partial \omega_2}\dot{\omega}_s+\cdots+\sum_{s=1}^{n}\frac{\partial I}{\partial \overset{(2\delta-2)}{\omega}_s}\overset{(2\delta-1)}{\omega}_s=0, \tag{19}$$

将系统的运动微分方程(3)的两端乘以 $\bar{\xi}_s=\xi_s-\omega_s\tau$,并对 s 求和,有

$$\sum_{s=1}^{n}\left[\frac{\partial \tilde{L}}{\partial \pi_s}+\sum_{\alpha=1}^{\delta}(-1)^{\alpha}\frac{\mathrm{d}^{\alpha}}{\mathrm{d}t^{\alpha}}\left(\frac{\partial \tilde{L}}{\partial \overset{(\alpha-1)}{\omega}_s}\right)-\tilde{R}_s\right]\bar{\xi}_s=0, \tag{20}$$

将(19)式与(20)式相加,分出含 $\overset{(2\delta-1)}{\omega}_k$ 的项,令其系数为零,得到

$$\sum_{s=1}^{n}(-1)^{\delta}\frac{\partial^2 \tilde{L}}{\partial \overset{(\delta-1)}{\omega}_s\,\partial \overset{(\delta-1)}{\omega}_k}\bar{\xi}_s+\frac{\partial I}{\partial \overset{(2\delta-2)}{\omega}_k}=0,$$
$$(k=1,2,\cdots,n). \tag{21}$$

由此解得

$$\bar{\xi}_s=\sum_{k=1}^{n}-\tilde{h}_{sk}\frac{\partial I}{\partial \overset{(2\delta-2)}{\omega}_k},\quad (s,k=1,2,\cdots,n), \tag{22}$$

其中 $\tilde{h}_{sk}$ 由下式确定:

$$\sum_{k=1}^{n}\tilde{h}_{sk}h_{kr}=\delta_{sr},\ h_{sk}=(-1)^{\delta}\frac{\partial^2 \tilde{L}}{\partial \overset{(\delta-1)}{\omega}_s\,\partial \overset{(\delta-1)}{\omega}_k}. \tag{23}$$

其次,令初积分(18)等于守恒量(13),即

$$I=\tilde{L}\tau+\sum_{s=1}^{n}\sum_{\alpha=1}^{\delta}\tilde{p}_{s/\alpha}\frac{\mathrm{d}^{\alpha-1}}{\mathrm{d}t^{\alpha-1}}(\xi_s-\omega_s\tau)+\lambda, \tag{24}$$

则有

$$\tau=\frac{1}{\tilde{L}}\left[I-\sum_{s=1}^{n}\sum_{\alpha=1}^{\delta}\tilde{p}_{s/\alpha}\frac{\mathrm{d}^{\alpha-1}}{\mathrm{d}t^{\alpha-1}}(\xi_s-\omega_s\tau)-\lambda\right]. \tag{25}$$

定理 2　如果准坐标下广义非保守系统(3)有一个初积分(13),则与此积分相应的积分因子 ξ_s 和函数 τ,λ 由关系式(22)和(25)确定.

6. 举　　例

已知广义力学系统的 Lagrange 函数为

$$L=\frac{1}{2}\dot{q}_1^2+\frac{1}{2}q_1^2\dot{q}_2^2+\frac{1}{2}\ddot{q}_1^2, \tag{26}$$

并设非势广义力为零,试求系统的守恒量.

解　首先研究正问题,求系统的守恒量.

选取准速度

$$\omega_1=\dot{q}_1,\ \omega_2=\frac{1}{2}q_1^2\dot{q}_2, \tag{27}$$

则有

$$\dot{q}_1=\omega_1,\ \dot{q}_2=\frac{2\omega_2}{q_1^2}, \tag{28}$$

于是

$$a_{11}=1,a_{12}=0,a_{21}=0,a_{22}=\frac{q_1^2}{2}, \tag{29}$$

$$b_{11}=1,b_{12}=0,b_{21}=0,b_{22}=\frac{2}{q_1^2}, \tag{30}$$

计算 Boltzmann 三标记号,得

$$\gamma_{12}^2=-\gamma_{21}^2=\frac{2}{q_1}, \tag{31}$$

其余为零.又

$$\tilde{L}=\frac{1}{2}\omega_1^2+\frac{2\omega_2^2}{q_1^2}+\frac{1}{2}\dot{\omega}_1^2, \tag{32}$$

$$\frac{\partial \tilde{L}}{\partial \pi_1}=\sum_{s=1}^{2}\frac{\partial \tilde{L}}{\partial q_s}b_{s1}=-\frac{4\omega_2^2}{q_1^3},$$

$$\frac{\partial \tilde{L}}{\partial \pi_2}=\sum_{s=1}^{2}\frac{\partial \tilde{L}}{\partial q_s}b_{s2}=0,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \tilde{L}}{\partial \omega_1}=\dot{\omega}_1,\ \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \tilde{L}}{\partial \omega_2}=\frac{4q_1\dot{\omega}_2-8\omega_1\omega_2}{q_1^3} \tag{33}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\frac{\partial \tilde{L}}{\partial \dot{\omega}_1}\right)=\dddot{\omega}_1,\ \frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\frac{\partial \tilde{L}}{\partial \dot{\omega}_2}\right)=0. \tag{34}$$

已知非势广义力为零,有

$$\tilde{Q}_1=\tilde{Q}_2=0, \tag{35}$$

$$\tilde{R}_1=-\tilde{Q}_1+\sum_{l=1}^{2}\sum_{k=1}^{2}\gamma_{k1}^{l}\left(\frac{\partial \tilde{L}}{\partial \omega_l}-\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \tilde{L}}{\partial \dot{\omega}_l}\right)\omega_k=-\frac{8\omega_2^2}{q_1^3},$$
$$\tilde{R}_2=-\tilde{Q}_2+\sum_{l=1}^{2}\sum_{k=1}^{2}\gamma_{k2}^{l}\left(\frac{\partial \tilde{L}}{\partial \omega_l}-\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \tilde{L}}{\partial \dot{\omega}_l}\right)\omega_k=\frac{8\omega_1\omega_2}{q_1^3}, \tag{36}$$

必要条件(15)给出

$$-\frac{4\omega_2^2}{q_1^3}\xi_1+\omega_1(\dot{\xi}_1-\omega_1\dot{\tau})+\frac{4\omega_2}{q_1^2}(\dot{\xi}_2-\omega_2\dot{\tau})+\dot{\omega}_1(\ddot{\xi}_1-2\dot{\omega}_1\dot{\tau}-\omega_1\ddot{\tau})+\left(\frac{1}{2}\omega_1^2+\frac{2\omega_2^2}{q_1^2}+\frac{1}{2}\dot{\omega}_1^2\right)\dot{\tau}$$
$$+\frac{8\omega_2^2}{q_1^3}(\xi_1-\omega_1\tau)-\frac{8\omega_1\omega_2}{q_1^3}(\xi_2-\omega_2\tau)+\dot{\lambda}+\tilde{\Phi}=0, \tag{37}$$

$$\tilde{\Phi}=-\mu_1\left(\frac{4\omega_2^2}{q_1^3}-\dot{\omega}_1+\dddot{\omega}_1\right)+\mu_2\frac{4\dot{\omega}_2}{q_1^2}. \tag{38}$$

当取

$$\mu_1=0,\mu_2=0,\xi_1=0,\xi_2=0,\tau=-1 \tag{39}$$

时,由(37)式和(38)式,得

$$\dot{\lambda}=0,取\ \lambda=0, \tag{40}$$

再利用(10)式求系统的广义动量

$$\tilde{p}_{1/1} = \frac{\partial \tilde{L}}{\partial \omega_1} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \tilde{L}}{\partial \dot{\omega}_1} = \omega_1 - \ddot{\omega}_1,$$

$$\tilde{p}_{1/2} = \frac{\partial \tilde{L}}{\partial \dot{\omega}_1} = \dot{\omega}_1,$$

$$\tilde{p}_{2/1} = \frac{\partial \tilde{L}}{\partial \omega_2} = \frac{4\omega_2}{q_1^2}. \tag{41}$$

守恒量(13)式给出

$$I = \frac{1}{2}\omega_1^2 - \omega_1\ddot{\omega}_2 + \frac{2\omega_2^2}{q_1^2} + \frac{1}{2}\dot{\omega}_1^2 = \text{const}. \tag{42}$$

其次,研究系统的逆问题.

由(23)式得

$$h_{11} = 1, h_{22} = \frac{4}{q_1^2}, h_{12} = h_{21} = 0. \tag{43}$$

故有 $\tilde{h}_{11} = 1, \tilde{h}_{22} = \frac{q_1^2}{4}$,其余为零.

利用(22)式求得

$$\bar{\xi}_1 = \omega_1, \bar{\xi}_2 = \omega_2, \tag{44}$$

亦即

$$\bar{\xi}_1 = \omega_1 = \xi_1 - \omega_1\tau,$$
$$\bar{\xi}_2 = \omega_2 = \xi_2 - \omega_2\tau, \tag{45}$$

又由(25)式,得

$$\tau = -1. \tag{46}$$

考虑到(45)式和(46)式,有

$$\xi_1 = 0, \xi_2 = 0. \tag{47}$$

[1] Noether A E 1918 *Nachr. Akad. Wiss. Phys.* **2** 235

[2] Lutzky M 1979 *J. Phys. A: Math. Gen.* **12** 973

[3] Mei F X 2000 *J. Beijing Inst. Technology.* **9** 120(in Chinese)[梅凤翔 2000 北京理工大学学报 **9** 120]

[4] Hojman S A 1992 *J. Phys. A: Math. Gen.* **25** L291

[5] Qiao Y F 1990 *Acta Mech. Sin.* **22** 99 (in Chinese)[乔永芬 1990 力学学报 **22** 99]

[6] Qiao Y F, Yue Q W 1993 *Chin, Sci. Bull.* **38** 314

[7] Qiao Y F, Yue Q W, Dong Y A 1994 *Appl. Math. Mech.* **15** 877

[8] Yue Q W, Dong Y A, Qiao Y F 1994 *Acta. Mech. Solida. Sin.* **15** 256 (in Chinese) [岳庆文、董永安、乔永芬 1994 固体力学学报 **15** 256]

[9] Mei F X 1999 *Applications of Lie Groups and Lie Algebras to Constrained Mechanical Systems* (Beijing: Science Prass) p30 (in Chinese) [梅凤翔 1999 李群和李代数对约束力学系统的应用(北京 科学出版社)第 30 项]

[10] Zhang Y, Shang M, Mei F X 2000 *Chin. Phys.* **9** 401

[11] Zhang Y 2001 *Acta Phys. Sin.* **50** 2059 (in Chinese) [张 毅 2001 物理学报 **50** 2059]

[12] Qiao Y F, Zhao S H 2001 *Acta Phys. Sin.* **50** 1 (in Chinese) [乔永芬、赵淑红 2001 物理学报 **50** 1]

[13] Mei F X 2003 *Acta Phys. Sin.* **52** 1048 (in Chinese) [梅凤翔 2003 物理学报 **52** 1048]

[14] Zhang Y 2003 *Acta Phys. Sin.* **52** 1832 (in Chinese)[张 毅 2003 物理学报 **52** 1832]

[15] Zhang Y, Mei F X 2003 *Acta Phys. Sin.* **52** 2368 (in Chinese) [张 毅、梅凤翔 2003 物理学报 **52** 2368]

[16] Qiao Y F, Li R J, Zhao S H 2005 *Chin. Phys.* **14** 882

[17] Djukic D S, Sutela T 1984 *Int. J. Nonlin. Mech.* **19** 331

[18] Qiao Y F, Zhang Y L, Han G C 2002 *Chin. Phus.* **11** 988

[19] Li R J, Qiao Y F, Zhao S H 2002 *Chin. Phys.* **11** 760

[20] Qiao Y F, Zhang Y L, Zhao S H 2002 *Acta Phys. Sin.* **51** 1661 (in Chinese) [乔永芬、张耀良、赵淑红 2002 物理学报 **51** 1661]

[21] Zhang Y , Ge W K 2003 *Acta Phys. Sin.* **52** 2363 (in Chinese) [张 毅、葛伟宽 2003 物理学报 **52** 2363]

[22] Zhang H B, Chen L Q, Liu Y W, Gu S L 2005 *Acta Phys. Sin.* **54** 2498 (in Chinese) [张宏彬、陈立群、刘荣万、顾书龙 2005 物理学报 **54** 2498]

Integrating factors and conservation theorems of Lagrange's equations for generalized nonconservative systems in terms of quasi-coordinates*

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Abstract

The conservation theorems of the Lagrange's equations for generalized nonconservative systems in terms of quasi-coordinates are studied by using the method of integrating factors. The differential equations of motion of systems are written. The definition of integrating factors is given. The necessary conditions for the existence of the conserved quantity are studied in detail. Finally, the conservation theorem and its inverse are established, and an example is given to illustrate the application of the result.

Keywords: quasi-coordinate, Lagrange's equation, integrating factor, conserved quantity

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