第 56 卷 第 2 期 2007 年 2 月
1000-3290/2007/56(02)/0627-10

物 理 学 报
ACTA PHYSICA SINICA

Vol.56,No.2,February,2007

辅助方程构造(2+1)维 Hybrid-Lattice 系统和离散的 $mKdV$ 方程的精确解*

套格图桑† 斯仁道尔吉

(内蒙古师范大学数学科学学院,呼和浩特 010022)

(2006 年 5 月 3 日收到;2006 年 6 月 19 日收到修改稿)

给出双曲函数型辅助方程和函数变换相结合的一种方法,借助符号计算系统 Mathematica 构造了(2+1)维 Hybrid-Lattice 系统和离散的 $mKdV$ 方程新的精确孤波解和三角函数波解.

关键词:辅助方程,函数变换,(2+1)维 Hybrid-Lattice 系统,离散的 $mKdV$ 方程

PACC:0230,0340,0290

1. 引 言

在构造非线性发展方程(组)精确解领域中有许多直接方法,比如:齐次平衡法[1]、双曲正切函数法[2]、Jacobi 椭圆函数展开法[3]、辅助方程法[4—11]、试探函数法[12]等.这些方法在求非线性离散系统精确解领域当中的应用比较少.文献[13—17]用试探函数法得到了(2+1)维 Hybrid-Lattice 系统和离散的 $mKdV$ 方程等,并且得到非线性离散系统的 Jacobi 椭圆函数精确解、精确孤波解和三角函数波解.本文进一步研究文献[7]给出的双曲函数型辅助方程,得到了该方程更多的新解,并给出该双曲函数型辅助方程和函数变换相结合的一种方法,借助符号计算系统 Mathematica 构造了(2+1)维 Hybrid-Lattice 系统和离散的 $mKdV$ 方程的新的精确孤波解和三角函数波解.

2. 双曲函数型辅助方程及其解

在文献[7]的基础上进一步研究得到了下列双曲函数型辅助方程:

$$\frac{\mathrm{d}z(\xi)}{\mathrm{d}\xi}=a+b\cosh(z(\xi)), \tag{1}$$

通过函数变换 $\mathrm{e}^{z(\xi)}=v(\xi)$ 和双曲函数的定义,我们从辅助方程(1)可以确定下列式子((2)式).用变量分离法,得到 $v(\xi)$,

$$\begin{aligned}\sinh(z(\xi))&=\frac{1}{2}(\mathrm{e}^{z(\xi)}-\mathrm{e}^{-z(\xi)})\\&=\frac{v^2(\xi)-1}{2v(\xi)},\\\cosh(z(\xi))&=\frac{1}{2}(\mathrm{e}^{z(\xi)}+\mathrm{e}^{-z(\xi)})\\&=\frac{v^2(\xi)+1}{2v(\xi)}.\end{aligned} \tag{2}$$

(A)双曲函数解

$$v(\xi)=\frac{-a-\sqrt{a^2-b^2}\tanh\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)}{b},\quad bv(\xi)\neq 0; \tag{3}$$

$$v(\xi)=\frac{-a-\sqrt{a^2-b^2}\coth\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)}{b},\quad bv(\xi)\neq 0; \tag{4}$$

* 国家自然科学基金(批准号:10461006)、内蒙古高等学校科学研究项目(批准号:NJ02035)、内蒙古自然科学基金(批准号:200408020103)和内蒙古师范大学青年研究项目基金(批准号:QN005023)资助的课题.

† E-mail:tgts@imnu.edu.cn

$$v(\xi)=\frac{-a-\sqrt{a^2-b^2}\,\mathrm{sech}(\sqrt{a^2-b^2}\,\xi)(\pm \mathrm{i}\pm\sinh(\sqrt{a^2-b^2}\,\xi))}{b},\quad bv(\xi)\neq 0; \tag{5}$$

$$v(\xi)=-\frac{2a+\sqrt{a^2-b^2}\left(\tanh\left(\frac{1}{4}\sqrt{a^2-b^2}\,\xi\right)+\coth\left(\frac{1}{4}\sqrt{a^2-b^2}\,\xi\right)\right)}{2b},\quad bv(\xi)\neq 0; \tag{6}$$

$$v(\xi)=\frac{-a}{b}+\frac{\sqrt{(a^2-b^2)(M^2+N^2)}-\sqrt{a^2-b^2}\,M\cosh(\sqrt{a^2-b^2}\,\xi)}{b(N+M\sinh(\sqrt{a^2-b^2}\,\xi))},$$
$$N+M\sinh(\sqrt{a^2-b^2}\,\xi)\neq 0,bv(\xi)\neq 0; \tag{7}$$

$$v(\xi)=\frac{-a}{b}-\frac{\sqrt{(a^2-b^2)(-M^2+N^2)}+\sqrt{a^2-b^2}\,M\sinh(\sqrt{a^2-b^2}\,\xi)}{b(N+M\cosh(\sqrt{a^2-b^2}\,\xi))},$$
$$N+M\cosh(\sqrt{a^2-b^2}\,\xi)\neq 0,bv(\xi)\neq 0; \tag{8}$$

$$v(\xi)=\frac{b\cosh\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)}{-a\cosh\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)+\sqrt{a^2-b^2}\sinh\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)},$$
$$v(\xi)\neq 0,-a\cosh\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)+\sqrt{a^2-b^2}\sinh\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)\neq 0; \tag{9}$$

$$v(\xi)=\frac{b\sinh\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)}{-a\sinh\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)+\sqrt{a^2-b^2}\cosh\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)},$$
$$v(\xi)\neq 0,-a\sinh\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)+\sqrt{a^2-b^2}\cosh\left(\frac{1}{2}\sqrt{a^2-b^2}\,\xi\right)\neq 0; \tag{10}$$

$$v(\xi)=\frac{b\cosh(\sqrt{a^2-b^2}\,\xi)}{-(a\cosh(\sqrt{a^2-b^2}\,\xi)\pm \mathrm{i}\sqrt{a^2-b^2})+\sqrt{a^2-b^2}\sinh(\sqrt{a^2-b^2}\,\xi)},$$
$$v(\xi)\neq 0,-(a\cosh(\sqrt{a^2-b^2}\,\xi)\pm \mathrm{i}\sqrt{a^2-b^2})+\sqrt{a^2-b^2}\sinh(\sqrt{a^2-b^2}\,\xi)\neq 0; \tag{11}$$

其中 $a^2-b^2>0$，$-M^2+N^2>0$，M，N 是不全为零任意常数.

(B)三角函数解

$$v(\xi)=\frac{-a+\sqrt{a^2+b^2}\tan\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)}{b},\quad bv(\xi)\neq 0; \tag{12}$$

$$v(\xi)=\frac{-a-\sqrt{-a^2+b^2}\cot\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)}{b},\quad bv(\xi)\neq 0; \tag{13}$$

$$v(\xi)=\frac{-a+\sqrt{-a^2+b^2}(\sec(\sqrt{-a^2+b^2}\,\xi)+\tan(\sqrt{-a^2+b^2}\,\xi))}{b},\quad bv(\xi)\neq 0; \tag{14}$$

$$v(\xi)=\frac{-2a+\sqrt{-a^2+b^2}\left(\tan\left(\frac{1}{4}\sqrt{-a^2+b^2}\,\xi\right)-\cot\left(\frac{1}{4}\sqrt{-a^2+b^2}\,\xi\right)\right)}{2b},\quad bv(\xi)\neq 0; \tag{15}$$

$$v(\xi)=\frac{-a}{b}+\frac{\pm\sqrt{(-a^2+b^2)(M^2-N^2)}-\sqrt{-a^2+b^2}\,M\cos(\sqrt{-a^2+b^2}\,\xi)}{b(N+M\sin(\sqrt{-a^2+b^2}\,\xi))},$$
$$N+M\sin(\sqrt{-a^2+b^2}\,\xi)\neq 0,bv(\xi)\neq 0; \tag{16}$$

$$v(\xi)=\frac{-a}{b}+\frac{\pm\sqrt{(-a^2+b^2)(M^2-N^2)}+\sqrt{-a^2+b^2}\,M\sin(\sqrt{-a^2+b^2}\,\xi)}{b(N+M\cos(\sqrt{-a^2+b^2}\,\xi))},$$
$$N+M\cos(\sqrt{-a^2+b^2}\,\xi)\neq 0,bv(\xi)\neq 0; \tag{17}$$

$$v(\xi)=-\frac{b\cos\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)}{a\cos\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)+\sqrt{-a^2+b^2}\sin\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)},$$

$$v(\xi)\neq 0,a\cos\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)+\sqrt{-a^2+b^2}\sin\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)\neq 0;\quad(18)$$

$$v(\xi)=\frac{b\sin\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)}{-a\sin\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)+\sqrt{-a^2+b^2}\cos\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)},$$

$$v(\xi)\neq 0,-a\sin\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)+\sqrt{-a^2+b^2}\cos\left(\frac{1}{2}\sqrt{-a^2+b^2}\,\xi\right)\neq 0;\quad(19)$$

$$v(\xi)=\frac{-b\cos(\sqrt{-a^2+b^2}\,\xi)}{a\cos(\sqrt{-a^2+b^2}\,\xi)\pm\sqrt{-a^2+b^2}+\sqrt{-a^2+b^2}\sin(\sqrt{-a^2+b^2}\,\xi)},$$

$$(v(\xi)\neq 0,a\cos(\sqrt{-a^2+b^2}\,\xi)\pm\sqrt{-a^2+b^2}+\sqrt{-a^2+b^2}\sin(\sqrt{-a^2+b^2}\,\xi)\neq 0);\quad(20)$$

其中 $-a^2+b^2>0,-M^2+N^2>0,M,N$ 是不全为零任意常数.

(C)指数函数解

$$v(\xi)=e^{a\xi},\quad a\neq 0,b=0.\qquad(21)$$

(D)有理解

$$v(\xi)=-1-\frac{2}{b\xi},\quad a=b\neq 0,\xi\neq 0.\qquad(22)$$

3. (2+1)维 Hybrid-Lattice 系统和离散的 $mKdV$ 方程的精确解

例 3.1　(2+1)维 Hybrid-Lattice 系统

$$\frac{\partial u_n(t)}{\partial t}=(1+\alpha u_n+\beta u_n^2)(u_{n-1}-u_{n+1}),\qquad(23)$$

其中 α,β 为常数.把 $u_n(n,x,t)=u_n(\xi),u_{n+1}(n,x,t)=u_{n+1}(\xi),u_{n-1}(n,x,t)=u_{n-1}(\xi),\xi=dn+\lambda x+\omega t$(其中 d,λ,ω 为常数)代入(23)式,后得到下列方程:

$$\omega u_n'(\xi)=(1+\alpha u_n(\xi)+\beta u_n^2(\xi))\times(u_{n-1}(\xi)-u_{n+1}(\xi)),\qquad(24)$$

在方程(24)中取下列形式的函数变换:

$$u_n(\xi)=g_0+g_1\sinh(z(\xi))+f_1\cosh(z(\xi)),\qquad(25)$$

$$\begin{aligned}u_{n+1}(\xi)=g_0+g_1(&\sinh(z(\xi))\cosh(d)\\&-\sinh(d)\cosh(z(\xi))+d+1)\\&+f_1(\sinh(z(\xi))\cosh(d)\\&+\sinh(d)\cosh(z(\xi))+d+1)),\end{aligned}\qquad(26)$$

$$\begin{aligned}u_{n-1}(\xi)=g_0+g_1(&\sinh(z(\xi))\cosh(d)\\&+\sinh(d)\cosh(z(\xi))+d)\\&+f_1(\sinh(z(\xi))\cosh(d)\\&-\sinh(d)\cosh(z(\xi))+d)),\end{aligned}\qquad(27)$$

将(1),(25)—(27)式一起代入(24)式,并令 $\cosh^p(z(\xi))\sinh^q(z(\xi)),(p=0,1;q=1,2,3)$ 的系数为零后得到如下非线性代数方程组:

$$(4\beta f_1^2g_1-4\beta f_1g_1^2)\sinh(d)=0,$$

$$\begin{aligned}&\beta f_1^3+b\omega g_1+\beta f_1^2g_1+\beta f_1g_1^2\\&+\beta g_1^3+(2\alpha f_1^2+4\beta f_1^2g_0\\&-2\alpha f_1g_1-4\beta f_1g_0g_1)\sinh(d)=0,\end{aligned}$$

$$(2\beta f_1^3-2\beta f_1^2g_1+2\beta f_1g_1^2-2\beta g_1^3)\sinh(d)=0,$$

$$\begin{aligned}&awf_1+\alpha f_1g_1+2\beta f_1g_1g_0+\alpha g_1^2\\&+2\beta g_0g_1^2+(4\beta f_1^2g_1-4\beta f_1g_1^2)\sinh(d)=0,\end{aligned}$$

$$\begin{aligned}&bwf_1+2\beta f_1^2g_1+2\beta f_1g_1^2\\&+(2\alpha f_1g_1+4\beta f_1g_0g_1-2\alpha g_1^2\\&-4\beta g_0g_1^2)\sinh(d)=0,\end{aligned}$$

$$\begin{aligned}&f_1+\beta f_1^3+\alpha f_1g_0+\beta f_1g_0^2\\&+g_1+bwg_1+\beta f_1^2g_1+\alpha g_0g_1\\&+\beta g_0^2g_1+(2\alpha f_1^2+4\beta f_1^2g_0-2\alpha f_1g_1\\&-4\beta f_1g_0g_1)\sinh(d)=0,\end{aligned}$$

$$\begin{aligned}&\alpha f_1^2+2\beta f_1^2g_0+awg_1+\alpha f_1g_1\\&+2\beta f_1g_0g_1+(2f_1+2\beta f_1^3+2\alpha f_1g_0\\&+2\beta f_1g_0^2-2g_1-2\beta f_1^2g_1\\&-2\alpha g_0g_1-2\beta g_0^2g_1)\sinh(d)=0.\end{aligned}$$

用符号计算系统 Mathematica 求出该方程组的如下解:

$$a=\pm\frac{2f_1\sqrt{\alpha^2-4\beta+4\beta^2f_1^2}}{w},$$

$$b=-\frac{4\beta f_1^2}{w},$$

$$g_0 = \frac{-\alpha \pm \sqrt{\alpha^2 - 4\beta + 4\beta^2 f_1^2}}{2\beta},$$

$$g_1 = f_1; \tag{28}$$

把(28),(2)式分别与(3)—(20)式一起代入(25)式后得到(2+1)维 Hybrid-Lattice 系统的下列形式的精确解:

(A)精确孤波解

$$u_{n(1,2)}(n,x,t) = \frac{-\alpha}{2\beta} \pm \frac{\sqrt{\alpha^2-4\beta}}{2\beta}\tanh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right);$$

$$u_{n(3,4)}(n,x,t) = \frac{-\alpha}{2\beta} \pm \frac{\sqrt{\alpha^2-4\beta}}{2\beta}\coth\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right);$$

$$u_{n(5,6)}(n,x,t) = \frac{-\alpha}{2\beta} \pm \frac{\sqrt{\alpha^2-4\beta}}{2\beta}\left(\mathrm{i}\ \mathrm{sech}\left(\frac{2\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) + \tanh\left(\frac{2\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\right);$$

$$u_{n(7,8)}(n,x,t) = \left(\pm\frac{\sqrt{\alpha^2-4\beta}}{4\beta}\cosh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) - \frac{\alpha}{4\beta}\sinh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\right)$$
$$\times \mathrm{csch}\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{2\sqrt{w^2}}\xi\right)\mathrm{sech}\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{2\sqrt{w^2}}\xi\right);$$

$$u_{n(9,10)}(n,x,t) = -\frac{\Xi \mp M\sqrt{\alpha^2-4\beta}\cosh\left(\frac{2\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) + M\alpha\sinh\left(\frac{2\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)}{2\beta\left(N + M\sinh\left(\frac{2\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\right)},$$
$$N + M\sinh\left(\frac{2\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) \neq 0;$$

$$u_{n(11,12)}(n,x,t) = -\frac{\Omega \mp M\sqrt{\alpha^2-4\beta}\sinh\left(\frac{2\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) + M\alpha\cosh\left(\frac{2\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)}{2\beta\left(N + M\cosh\left(\frac{2\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\right)},$$
$$N + M\cosh\left(\frac{2\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) \neq 0;$$

$$u_{n(13,14)}(n,x,t) = \frac{-(\mp\alpha^2 \pm 4\beta + \alpha\Delta)\cosh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) \pm \sqrt{\alpha^2-4\beta}(\mp\alpha \mid \Delta)\sinh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)}{2\beta\left(\Delta\cosh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) \pm \sqrt{(\alpha^2-4\beta)}\sinh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\right)},$$
$$\Delta\cosh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) \pm \sqrt{(\alpha^2-4\beta)}\sinh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) \neq 0;$$

$$u_{n(15,16)}(n,x,t) = \frac{-(\mp\alpha^2 \pm 4\beta \pm \alpha\Delta)\sinh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) \pm \sqrt{\alpha^2-4\beta}(\alpha \pm \Delta)\cosh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)}{2\beta\left(\pm\Delta\sinh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) \pm \sqrt{(\alpha^2-4\beta)}\cosh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\right)},$$
$$\pm\Delta\sinh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) \pm \sqrt{(\alpha^2-4\beta)}\cosh\left(\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right) \neq 0;$$

$$u_{n(17,18)}(n,x,t)=-\frac{\Gamma\cosh\left(2\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha^2-4\beta}(\alpha\mp\Delta)\left(\pm\mathrm{i}+\sinh\left(2\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\right)}{2\beta\left(\pm\Delta\cosh\left(2\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{(\alpha^2-4\beta)}\left(\pm\mathrm{i}+\sinh\left(2\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\right)\right)},$$

$$\pm\Delta\cosh\left(2\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{(\alpha^2-4\beta)}\left(\pm\mathrm{i}+\sinh\left(2\frac{\sqrt{(\alpha^2-4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\right)\neq 0;$$

其中

$$\Xi=N\alpha\pm\sqrt{(M^2+N^2)(\alpha^2-4\beta)},$$

$$\Delta=\sqrt{\alpha^2+4\beta(-1+\beta f_1^2)},$$

$$\Gamma=-\alpha^2+4\beta\pm\alpha\Delta,$$

$$\Omega=N\alpha\pm\sqrt{(-M^2+N^2)(\alpha^2-4\beta)},\alpha^2-4\beta>0,\alpha^2+4\beta(-1+\beta f_1^2)>0,\xi=dn+\lambda x+wt.$$

(B)三角函数波解

$$u_{n(19,20)}(n,x,t)=\frac{-\alpha}{2\beta}\pm\frac{\sqrt{-\alpha^2+4\beta}}{2\beta}\tan\left(\frac{\sqrt{(-\alpha^2+4\beta)f_1^2}}{\sqrt{w^2}}\xi\right);$$

$$u_{n(21,22)}(n,x,t)=\frac{-\alpha}{2\beta}\pm\frac{\sqrt{-\alpha^2+4\beta}}{2\beta}\cot\left(\frac{\sqrt{(-\alpha^2+4\beta)f_1^2}}{\sqrt{w^2}}\xi\right);$$

$$u_{n(23,24)}(n,x,t)=\frac{-\alpha}{2\beta}\pm\frac{\sqrt{-\alpha^2+4\beta}}{2\beta}\left(\pm\sec\left(\frac{\sqrt{(-\alpha^2+4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)+\tan\left(\frac{\sqrt{(-\alpha^2+4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\right);$$

$$u_{n(25,26)}(n,x,t)=\frac{1}{4\beta}\left(\pm\sqrt{-\alpha^2+4\beta}\cos\left(\frac{\sqrt{(-\alpha^2+4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)-\alpha\sin\left(\frac{\sqrt{(-\alpha^2+4\beta)f_1^2}}{\sqrt{w^2}}\xi\right)\right)$$
$$\times\csc\left(\frac{\sqrt{(-\alpha^2+4\beta)f_1^2}}{2\sqrt{w^2}}\xi\right)\sec\left(\frac{\sqrt{(-\alpha^2+4\beta)f_1^2}}{2\sqrt{w^2}}\xi\right);$$

$$u_{n(27,28)}(n,x,t)=\frac{-\alpha\pm\sqrt{\alpha^2+4\beta(-1+\beta f_1^2)}}{2\beta}$$
$$+\frac{f_1\left(a+\frac{\pm\sqrt{(-a^2+b^2)(M^2-N^2)}+M\sqrt{-a^2+b^2}\cos(\sqrt{-a^2+b^2}\xi)}{N+M\sin(\sqrt{-a^2+b^2}\xi)}\right)^2}{2b\left(-a+\frac{\sqrt{(-a^2+b^2)(M^2-N^2)}-M\sqrt{-a^2+b^2}\cos(\sqrt{-a^2+b^2}\xi)}{N+M\sin(\sqrt{-a^2+b^2}\xi)}\right)},$$

$$-a+\frac{\sqrt{(-a^2+b^2)(M^2-N^2)}-M\sqrt{-a^2+b^2}\cos(\sqrt{-a^2+b^2}\xi)}{N+M\sin(\sqrt{-a^2+b^2}\xi)}\neq 0;$$

$$u_{n(29,30)}(n,x,t)=\frac{-N(b\alpha+2a\beta f_1\mp b\sqrt{\alpha^2+4\beta(-1+\beta f_1^2)})\mp 2\beta f_1\sqrt{(-a^2+b^2)(M^2-N^2)}}{2b\beta(N+M\cos(\sqrt{-a^2+b^2}\xi))}$$
$$-\frac{M(b\alpha+2a\beta f_1\mp b\sqrt{\alpha^2+4\beta(-1+\beta f_1^2)})\cos(\sqrt{-a^2+b^2}\xi)-2\sqrt{-a^2+b^2}M\beta f_1\sin(\sqrt{-a^2+b^2}\xi)}{2b\beta(N+M\cos(\sqrt{-a^2+b^2}\xi))};$$

其中

$$b\beta(N+M\cos(\sqrt{-a^2+b^2}\xi))\neq 0,$$

$$a=\pm\frac{2f_1\sqrt{a^2-4\beta+4\beta^2f_1^2}}{w},$$

$$b=-\frac{4\beta f_1^2}{w}.$$

$$u_{n(31,32)}(n,x,t)=\frac{(-\alpha^2+4\beta\pm\alpha P)\cos(Q\xi)\pm\sqrt{-\alpha^2+4\beta}(\pm\alpha-P)\sin(Q\xi)}{2\beta(P\cos(Q\xi)\mp\sqrt{(-\alpha^2+4\beta)}\sin(Q\xi))},$$
$$P\cos(Q\xi)\mp\sqrt{(-\alpha^2+4\beta)}\sin(Q\xi)\neq 0;$$

$$u_{n(33,34)}(n,x,t)=\frac{\mp(\alpha\pm P)\sqrt{-\alpha^2+4\beta}\cos(Q\xi)\pm(\alpha^2-4\beta\mp\alpha P)\sin(Q\xi)}{2\beta(\pm P\sin(Q\xi)\pm\sqrt{(-\alpha^2+4\beta)}\cos(Q\xi))},$$
$$\pm P\sin(Q\xi)\pm\sqrt{(-\alpha^2+4\beta)}\cos(Q\xi)\neq 0;$$

$$u_{n(35,36)}(n,x,t)=\frac{R(\mp P\cos(2Q\xi)\pm\sqrt{-\alpha^2+4\beta})+(4\beta^2f_1^2\cos(2Q\xi)\pm\sqrt{-\alpha^2+4\beta}R\sin(2Q\xi))}{\beta((-P\cos(2Q\xi)\pm\sqrt{-\alpha^2+4\beta})+\sqrt{-\alpha^2+4\beta}\sin(2Q\xi))},$$
$$-P\cos(2Q\xi)\pm\sqrt{-\alpha^2+4\beta}+\sqrt{-\alpha^2+4\beta}\sin(2Q\xi)\neq 0;$$

其中

$$P=\sqrt{\alpha^2+4\beta(-1+\beta f_1^2)},$$
$$Q=\frac{\sqrt{(-\alpha^2+4\beta)f_1^2}}{\sqrt{w^2}},$$
$$R=-\alpha\pm P,$$
$$\alpha^2+4\beta(-1+\beta f_1^2)>0,$$
$$-\alpha^2+4\beta>0,$$
$$\xi=dn+\lambda x+wt.$$

当 $b=a$ 时,用符号计算系统 Mathematica 求出该方程组的如下解:

$$a=-\frac{4\beta f_1^2}{w},$$
$$g_0=\frac{-\alpha+2\beta f_1}{2\beta},$$
$$g_1=f_1,$$
$$\alpha^2=4\beta. \tag{29}$$

(C)有理解

把(22),(29),(2)式一起代入(25)式后得到(2+1)维 Hybrid-Lattice 系统的下列形式的有理解:

$$u_{n(37)}(n,x,t)=\frac{2(w-\alpha\xi f_1)}{\alpha^2\xi f_1},$$
$$f_1\xi\neq 0,\xi=dn+\lambda x+wt.$$

例 3.2 离散的 $mKdV$ 方程

$$\frac{\partial u_n(t)}{\partial t}=(\alpha-u_n^2)(u_{n+1}-u_{n-1}), \tag{30}$$

把 $u_n(n,t)=u_n(\xi),u_{n+1}(n,t)=u_{n+1}(\xi),u_{n-1}(n,t)=u_{n-1}(\xi),\xi=dn+wt$ 代入(30)式,后得到下列方程:

$$wu_n'(\xi)=(\alpha-u_n^2(\xi))(u_{n+1}(\xi)-u_{n-1}(\xi)), \tag{31}$$

在方程(31)中取下列形式的函数变换:

$$u_n(\xi)=g_0+g_1\sinh(z(\xi))+f_1\cosh(z(\xi)), \tag{32}$$

$$u_{n+1}(\xi)=g_0+g_1(\sinh(z(\xi))\cosh(d)-\sinh(d)\cosh(z(\xi))+d+1)+f_1(\sinh(z(\xi))\cosh(d)+\sinh(d)\cosh(z(\xi))+d+1)), \tag{33}$$

$$u_{n-1}(\xi)=g_0+g_1(\sinh(z(\xi))\cosh(d)+\sinh(d)\cosh(z(\xi))+d)+f_1(\sinh(z(\xi))\cosh(d)-\sinh(d)\cosh(z(\xi))+d)), \tag{34}$$

将(1),(32)—(34)式一起代入(31)式,并令 $\cosh^p(z(\xi))\sinh^q(z(\xi)),(p=0,1;q=1,2,3)$ 的系数为零后得到一非线性代数方程组(未列出),用符号计算系统 Mathematica 求出该方程组的如下解:

$$a=\pm\frac{4f_1\sqrt{\alpha+f_1^2}}{w},$$
$$b=-\frac{4\beta f_1^2}{w},$$
$$g_0=\pm\sqrt{\alpha+f_1^2},$$
$$g_1=f_1. \tag{35}$$

把(35),(2)式分别与(3)—(20)式一起代入(31)式后得到离散的 $mKdV$ 方程的下列形式精确解:

(A)精确孤波解

$$u_{n(38,39)}(n,t)=\pm\sqrt{\alpha}\tanh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right),\quad \alpha>0;$$

$$u_{n(40,41)}(n,t)=\pm\sqrt{\alpha}\coth\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right),\quad \alpha>0;$$

$$u_{n(42,43)}(n,t)=\pm\sqrt{\alpha}\left(\pm\mathrm{i}+\sinh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\right)\operatorname{sech}\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right),\quad \alpha>0;$$

$$u_{n(44,45)}(n,t)=\pm\frac{\sqrt{\alpha}}{2}\left(\tanh\left(\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)+\coth\left(\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\right),\quad \alpha>0;$$

$$u_{n(46,47)}(n,t)=-\frac{\pm\sqrt{\alpha(M^2+N^2)}\mp M\sqrt{\alpha}\cosh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{N+M\sinh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha>0,N+M\sinh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(48,49)}(n,t)=\frac{\pm\sqrt{\alpha(-M^2+N^2)}\pm M\sqrt{\alpha}\sinh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{N+M\cosh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha>0,-M^2+N^2>0,N+M\cosh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(50,51)}(n,t)=\frac{\alpha\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha(\alpha+f_1^2)}\sinh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{\sqrt{\alpha+f_1^2}\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha}\sinh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha>0,\sqrt{\alpha+f_1^2}\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha}\sinh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(52,53)}(n,t)=\frac{-\alpha\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha(\alpha+f_1^2)}\sinh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{\sqrt{\alpha+f_1^2}\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\mp\sqrt{\alpha}\sinh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha>0,\sqrt{\alpha+f_1^2}\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(54,55)}(n,t)=\frac{\alpha\sinh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha(\alpha+f_1^2)}\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{\sqrt{\alpha+f_1^2}\sinh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha}\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha>0,\sqrt{\alpha+f_1^2}\sinh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha}\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(56,57)}(n,t)=\frac{\alpha\sinh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\mp\sqrt{\alpha(\alpha+f_1^2)}\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{-\sqrt{\alpha+f_1^2}\sinh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha}\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha>0,\ -\sqrt{\alpha+f_1^2}\sinh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha}\cosh\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(58,59)}(n,t)=\frac{\alpha\cosh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha(\alpha+f_1^2)}\left(\pm\mathrm{i}+\sinh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\right)}{\sqrt{\alpha+f_1^2}\cosh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha}\left(\pm\mathrm{i}+\sinh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\right)},$$

$$\alpha>0,\sqrt{\alpha+f_1^2}\cosh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha}\left(\pm\mathrm{i}+\sinh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\right)\neq 0;$$

$$u_{n(60,61)}(n,t)=\frac{-\alpha\cosh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{\alpha(\alpha+f_1^2)}\left(\pm\mathrm{i}+\sinh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\right)}{\sqrt{\alpha+f_1^2}\cosh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\mp\sqrt{\alpha}\left(\pm\mathrm{i}+\sinh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\right)},$$

$$\alpha>0,\sqrt{\alpha+f_1^2}\cosh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\mp\sqrt{\alpha}\left(\pm\mathrm{i}+\sinh\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\right)\neq 0.$$

(B)三角函数波解

$$u_{n(62,63)}(n,t)=\pm\sqrt{-\alpha}\tan\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right),\quad \alpha<0;$$

$$u_{n(64,65)}(n,t)=\pm\sqrt{-\alpha}\cot\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right),\quad \alpha<0;$$

$$u_{n(66,67)}(n,t)=\mp\frac{\sqrt{-\alpha}\left(\cos\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)+\sin\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\right)}{\cos\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)-\sin\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha<0,\cos\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)-\sin\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(68,69)}(n,t)=\mp\frac{1}{2}\sqrt{-\alpha}\cos\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\csc\left(\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\sec\left(\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right),\quad \alpha<0;$$

$$u_{n(70,71)}(n,t)=\frac{\pm\sqrt{-\alpha(M^2-N^2)}\mp M\sqrt{-\alpha}\cos\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{N+M\sin\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha<0,N+M\sin\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(72,73)}(n,t)=\frac{\pm\sqrt{-\alpha(M^2-N^2)}\pm M\sqrt{-\alpha}\sin\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{N+M\cos\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha<0, N+M\cos\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(74,75)}(n,t)=\frac{-\alpha\cos\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha(\alpha+f_1^2)}\sin\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{-\sqrt{\alpha+f_1^2}\cos\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha}\sin\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha<0, -\sqrt{\alpha+f_1^2}\cos\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha}\sin\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(76,77)}(n,t)=\frac{-\alpha\cos\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\mp\sqrt{-\alpha(\alpha+f_1^2)}\sin\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{\sqrt{\alpha+f_1^2}\cos\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha}\sin\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha<0, \sqrt{\alpha+f_1^2}\cos\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha}\sin\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(78,79)}(n,t)=\frac{\alpha\sin\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha(\alpha+f_1^2)}\cos\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{\sqrt{\alpha+f_1^2}\sin\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha}\cos\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha<0, \sqrt{\alpha+f_1^2}\sin\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha}\cos\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(80,81)}(n,t)=\frac{\alpha\sin\left(2\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\mp\sqrt{-\alpha(\alpha+f_1^2)}\cos\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{-\sqrt{\alpha+f_1^2}\sin\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha}\cos\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha<0, -\sqrt{\alpha+f_1^2}\sin\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha}\cos\left(2\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(82,83)}(n,t)=\frac{-\alpha\cos\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha(\alpha+f_1^2)}\pm\sqrt{-\alpha(\alpha+f_1^2)}\sin\left(4\frac{\sqrt{\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{-\sqrt{\alpha+f_1^2}\cos\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha}\pm\sqrt{-\alpha}\sin\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha<0, -\sqrt{\alpha+f_1^2}\cos\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha}\pm\sqrt{-\alpha}\sin\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\neq 0;$$

$$u_{n(84,85)}(n,t)=\frac{-\alpha\cos\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\mp\sqrt{-\alpha(\alpha+f_1^2)}\mp\sqrt{-\alpha(\alpha+f_1^2)}\sin\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)}{\sqrt{\alpha+f_1^2}\cos\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)\pm\sqrt{-\alpha}\pm\sqrt{-\alpha}\sin\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right)},$$

$$\alpha < 0, \sqrt{\alpha + f_1^2}\cos\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right) \pm \sqrt{-\alpha} \pm \sqrt{-\alpha}\sin\left(4\frac{\sqrt{-\alpha f_1^2}}{\sqrt{w^2}}\xi\right) \neq 0,$$

其中 $\alpha + f_1^2 > 0, \xi = dn + wt$.

4. 结 论

辅助方程和函数变换相结合的方法,被广泛应用到连续的非线性发展方程求解领域.但是非线性离散系统求解领域当中的应用比较少.本文给出双曲函数型辅助方程和函数变换相结合的一种方法,并把该方法应用到求非线性离散系统的精确解领域中,借助符号计算系统 Mathematica 得到了(2+1)维 Hybrid-Lattice 系统和离散的 $mKdV$ 方程新的精确孤波解和三角函数波解.其中文献[13—17]没有得到下列形式的精确解:$u_{n(7,8)}(n,x,t)$ ~ $u_{n(17,18)}(n,x,t)$;$u_{n(23,24)}(n,x,t)$ ~ $u_{n(35,36)}(n,x,t)$;$u_{n(37)}(n,x,t)$,$u_{n(42,43)}(n,t)$ ~ $u_{n(60,61)}(n,t)$;$u_{n(66,67)}(n,t)$ ~ $u_{n(84,85)}(n,t)$.

[1] Wang M L 1995 *Phys*. *Lett*. A **199** 279

[2] Parkes E J, Duffy B R 1996 *Comp*. *Phys*. *Commun*. **98** 288

[3] Parkes E J, Duffy B D 1997 *Phys*. *Lett*. A **229** 217

[4] Sirendaoreji, Sun J 2003 *Phys*. *Lett*. A **309** 169

[5] Ean E G 2000 *Phys*. *Lett*. A **277** 212

[6] Chen Y, Li B, Zhabg H Q 2003 *Chin*, *Phys*. **12** 940

[7] Taogetusang, Sirendaoerji 2004 *Acta Phys*. *Sin*. **53** 4052 (in Chinese) [套格图桑、斯仁道尔吉 2004 物理学报 **53** 4052]

[8] Li D S, Zhang H Q 2003 *Acta Phys*. *Sin*. **52** 2373 (in Chinese [李德生、张鸿庆 2003 物理学报 **52** 2373]

[9] Fu Z T, Liu S K, Liu S D 2004 *Acta Phys*. *Sin*. **53** 343 (in Chinese) [李付遵涛、刘式适、刘式达 2004 物理学报 **53** 343]

[10] Wang Z, Li D S, Lu H F, Zhabg H Q 2005 *Chin*, *Phys*. **14** 2158

[11] Li D S, Zhabg H Q 2004 *Chin*. *Phys*. **13** 1377

[12] Yan Z Y, Zhang H Q 1999 *Acta Phys*. *Sin*. **48** 1957 (in Chinese) [闫振亚、张鸿庆 物理学报 1999 **48** 1957]

[13] Zhu J M, Ma Z Y, Zheng S L 2005 *Acta Phys*. *Sin*. **54** 483 (in Chinese) [朱加民、马正义、郑春龙 2005 物理学报 **54** 483]

[14] Yu Y X, Wang Q, Zhao X Q, Zhi H Y, Zahgn H Q 2005 *Acta*. *Phys*. *Sin*. **54** 3992 (in Chinese) [于亚璇、王 琪、赵雪芹、智红燕、张鸿庆 2005 物理学报 **54** 3992]

[15] Zhu J M 2005 *Chin*. *Phys*. **14** 1290

[16] Zhu J M, Ma Z Y 2005 *Chin*. *Phys*. **14** 17

[17] Zha Q L, Sirendaoerji 2006 *Chin*. *Phys*. **15** 475

Constructing the exact solutions of the (2+1)-dimensional Hybrid-Lattice and discrete $mKdV$ equation*

Taogetusang† Sirendaoerji

(*The College of Mathematical Science*, *Inner Mongolia Normal University*, *Huhhot* 010022, *China*)

(Received 3 May 2006; revised manuscript received 19 June 2006)

Abstract

The paper gives a method for combining the auxiliary equation with hyperbolic function by function transition. And then the method is applied to construct the new solitary wave solutions and the trigonomical function wave solutions to the (2+1)-dimensional Hybrid-Lattice and discrete $mKdV$ equation with the help of the symbolic computation system Mathematica.

Keywords: auxiliary equation, function transformation, (2+1)-dimensional Hybrid-Lattice, discrete $mKdV$ equation

PACC: 0230, 0340, 0290

* Project supported by the National Natural Science Foundation of China (Grant No. 10461006), the High Education Science Research Program of Inner Mongolia Autonomous Region (Grant No. NJ02035), Natural Science Foundation of Inner Mongolia Autonomous Region (Grant No. 200408020103) and the Youth Research Program of Inner Mongolia Normal University (Grant NO. QN005023).

† E-mail: tgts@imnu.edu.cn