第 56 卷 第 6 期 2007 年 6 月
1000-3290/2007/56(06)/3054-06

物 理 学 报
ACTA PHYSICA SINICA

Vol.56,No.6,June,2007

事件空间中完整系统的 Lie 对称性与绝热不变量*

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（2006 年 8 月 23 日收到；2006 年 10 月 14 日收到修改稿）

研究事件空间中完整力学系统 Lie 对称性的摄动与绝热不变量.基于力学系统的高阶绝热不变量的概念，研究在小扰动作用下系统 Lie 对称性的摄动，得到了事件空间中完整力学系统的一类 Hojman 形式的高阶绝热不变量，给出了绝热不变量存在的条件及形式.并举例说明结果的应用.

关键词：事件空间，对称性，摄动，绝热不变量

PACC：0320

1. 引 言

1960 年，Synge[1] 研究了事件空间中完整保守系统动力学.这种研究不仅具有几何意义，而且有重要的力学意义[2].动力学系统的对称性与守恒量的研究是分析力学的一个近代发展方向[3—27].关于事件空间中力学系统对称性和守恒量的研究已取得了一些重要成果[28—34].梅凤翔[28—30]研究了事件空间中完整系统的 Noether 对称性、Lie 对称性和 Mei 对称性及其相应的 Noether 守恒量；研究了事件空间中非完整系统的 Lie 对称性和 Noether 守恒量.李元成等[31—33]研究了事件空间中单面约束力学系统的 Noether 理论.许学军等[34]研究了事件空间中完整系统的对称性与 Hojman 守恒量.本文进一步在事件空间中研究受小扰动作用后完整约束力学系统 Lie 对称性的摄动，得到了一类 Hojman 形式的高阶绝热不变量，给出了绝热不变量存在的条件及其形式.

2. Lie 对称性与精确不变量

假设力学系统受有双面理想完整约束，其位形由 n 个广义坐标 $q_s\ (s=1,\cdots,n)$ 来确定.建立 $(n+1)$ 维扩充的位形空间（事件空间），此空间中点的坐标是广义坐标 $q_s\ (s=1,\cdots,n)$ 和时间 t，引入记号

$$x_1 = t,\ x_{s+1} = q_s,\quad (s=1,\cdots,n),\tag{1}$$

那么，所有变量 $x_\alpha\ (\alpha=1,\cdots,n+1)$ 可作为某参数 τ 的已知函数.令 $x_\alpha = x_\alpha(\tau)$ 是 c^2 类曲线，使得

$$\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau} = x'_\alpha,\quad (\alpha = 1,\cdots,n+1)\tag{2}$$

不同时为零，有

$$\dot{x}_\alpha = \frac{\mathrm{d}x_\alpha}{\mathrm{d}t} = \frac{x'_\alpha}{x'_1},\tag{3}$$

假设系统在位形空间中的 Lagrange 函数为 $L = L(t,\boldsymbol{q},\dot{\boldsymbol{q}})$，非势广义力为 $Q''_s = Q''_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})$，则事件空间中的 Lagrange 函数为

$$\Lambda(x_\alpha,x'_\alpha) = x'_1 L\left(x_\alpha,\frac{x'_2}{x'_1},\cdots,\frac{x'_{n+1}}{x'_1}\right),\tag{4}$$

定义事件空间中的非势广义力为[2]

$$P''_1 = -Q''_s x'_{s+1},\ P''_{s+1} = x'_1 Q''_s\left(x_\alpha,\frac{x'_2}{x'_1},\cdots,\frac{x'_{n+1}}{x'_1}\right),\tag{5}$$

则系统的运动微分方程可表为

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial\Lambda}{\partial x'_\alpha} - \frac{\partial\Lambda}{\partial x_\alpha} = P''_\alpha,\ (\alpha = 1,\cdots,n+1).\tag{6}$$

方程(6)中 $(n+1)$ 个方程不是彼此独立的，假设由方程(6)可解出后面 n 个 x''_{s+1}，有[28]

$$x''_{s+1} = h_{s+1}(x_\alpha,x'_\alpha,x''_1),\ (s = 1,\cdots,n).\tag{7}$$

引入参数 τ 和事件空间中坐标 x_α 的无限小群变换为[30]

$$\tau^* = \tau,\ x_1^*(\tau^*) = x_1(\tau),$$
$$x^*_{s+1}(\tau^*) = x_{s+1}(\tau) + \varepsilon\xi^0_{s+1}(x_\alpha,x'_\alpha),$$

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* 江苏省高校自然科学基金（批准号：04KJA130135）资助的课题.

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$$(s=1,\cdots,n),\tag{8}$$

其中 ε 为无限小参数,ξ_{s+1}^0 为无限小生成元.取无限小生成元向量

$$X_0^{(0)}=\xi_{s+1}^0\frac{\partial}{\partial x_{s+1}},\tag{9}$$

它的一次扩展

$$X_0^{(1)}=X_0^{(0)}+\frac{\bar{d}}{d\tau}\xi_{s+1}^0\frac{\partial}{\partial x_{s+1}'},\tag{10}$$

以及它的二次扩展

$$X_0^{(2)}=X_0^{(1)}+\frac{\bar{d}}{d\tau}\frac{\bar{d}}{d\tau}\xi_{s+1}^0\frac{\partial}{\partial x_{s+1}''},\tag{11}$$

其中

$$\frac{\bar{d}}{d\tau}=\frac{\partial}{\partial\tau}+x_\alpha'\frac{\partial}{\partial x_\alpha}+x_1''\frac{\partial}{\partial x_1'}+h_{s+1}\frac{\partial}{\partial x_{s+1}'},\tag{12}$$

方程(7)在无限小变换(8)下的不变性归为如下的 Lie 对称性确定方程

$$\frac{\bar{d}}{d\tau}\frac{\bar{d}}{d\tau}\xi_{s+1}^0=X_0^{(1)}(h_{s+1}).\tag{13}$$

在无限小变换(8)下,如果生成元 ξ_{s+1}^0 满足确定方程(13),且存在某函数 $\mu_0=\mu_0(x_\alpha,x'_\alpha)$ 使得

$$\frac{\partial h_{s+1}}{\partial x_{s+1}'}+\frac{\bar{d}}{d\tau}\ln\mu_0=0,\tag{14}$$

则事件空间中未受扰完整力学系统的 Lie 对称性直接导致 Hojman 守恒量,形如[34]

$$I_0=\frac{1}{\mu_0}\frac{\partial}{\partial x_{s+1}}(\mu_0\xi_{s+1}^0)+\frac{1}{\mu_0}\frac{\partial}{\partial x_{s+1}'}\left(\mu_0\frac{\bar{d}}{d\tau}\xi_{s+1}^0\right)=\mathrm{const}.\tag{15}$$

守恒量(15)是一个精确不变量,它揭示了事件空间中未受扰完整力学系统的 Lie 对称性与不变量之间的关系.

3. Lie 对称性的摄动与 Hojman 绝热不变量

定义 1　若 $I_z(x_\alpha,x_\alpha',\upsilon)$ 是事件空间中完整系统的一个含有小参数 υ 的最高次幂为 z 的物理量,其对参数 τ 的一阶导数正比于 υ^{z+1},则称 I_z 为事件空间中完整力学系统的 z 阶绝热不变量.

假设完整力学系统受到小扰动 $\upsilon W_s=W_s(t,q,\dot{q})$ 的作用,在事件空间中有

$$\upsilon R_1=-\upsilon W_s x_{s+1}',$$
$$\upsilon R_{s+1}=\upsilon x_1' W_s\left(x_\alpha,\frac{x_2'}{x_1'},\cdots,\frac{x_{n+1}'}{x_1'}\right),\tag{16}$$

则系统的运动微分方程(6)变为

$$\frac{d}{d\tau}\frac{\partial\Lambda}{\partial x_\alpha'}-\frac{\partial\Lambda}{\partial x_\alpha}=P_\alpha''+\upsilon R_\alpha,$$
$$(\alpha=1,\cdots,n+1).\tag{17}$$

方程(17)中的$(n+1)$个方程并不独立,展开方程(17)的后 n 个方程可得到

$$x_{s+1}''=h_{s+1}+\upsilon\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1},$$
$$(s,k=1,\cdots,n),\tag{18}$$

其中 $\Delta=\det\left(\frac{\partial^2\Lambda}{\partial x_{s+1}'\partial x_{k+1}'}\right)$,$\Delta_{s+1,k+1}$ 是行列式 Δ 中元素$\frac{\partial^2\Lambda}{\partial x_{s+1}'\partial x_{k+1}'}$的代数余子式.

在小扰动 $\upsilon R_\alpha(x_\beta,x_\beta')$ 的作用下,系统原有的对称性与不变量相应地会发生改变.假设扰动后的无限小生成元 ξ_{s+1} 是在系统无扰动的对称性变换生成元 ξ_{s+1}^0 基础上发生的小摄动,有

$$\xi_{s+1}=\xi_{s+1}^0+\upsilon\xi_{s+1}^1+\upsilon^2\xi_{s+1}^2+\cdots,\tag{19}$$

而无限小群变换(8)成为

$$\tau^*=\tau,x_1^*(\tau^*)=x_1(\tau),$$
$$x_{s+1}^*(\tau^*)=x_{s+1}(\tau)+\varepsilon\xi_{s+1}(x_\alpha,x_\alpha'),$$
$$(s=1,\cdots,n).\tag{20}$$

无限小生成元向量及其一次、二次扩展为

$$X^{(0)}=\xi_{s+1}\frac{\partial}{\partial x_{s+1}},\tag{21}$$

$$X^{(1)}=X^{(0)}+\frac{\tilde{d}}{d\tau}\xi_{s+1}\frac{\partial}{\partial x_{s+1}'},\tag{22}$$

$$X^{(2)}=X^{(1)}+\frac{\tilde{d}}{d\tau}\frac{\tilde{d}}{d\tau}\xi_{s+1}\frac{\partial}{\partial x_{s+1}''},\tag{23}$$

其中

$$\frac{\tilde{d}}{d\tau}=\frac{\partial}{\partial\tau}+x_\alpha'\frac{\partial}{\partial x_\alpha}+x_1''\frac{\partial}{\partial x_1'}+\left(h_{s+1}+\upsilon\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\right)\frac{\partial}{\partial x_{s+1}'}.\tag{24}$$

比较(24)式和(12)式,有

$$\frac{\tilde{d}}{d\tau}=\frac{\bar{d}}{d\tau}+\upsilon\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\frac{\partial}{\partial x_{s+1}'},\tag{25}$$

扰动后的运动方程(18)在无限小变换(20)下的不变性归为如下的 Lie 对称性确定方程

$$\frac{\tilde{d}}{d\tau}\frac{\tilde{d}}{d\tau}\xi_{s+1}=X^{(1)}(h_{s+1})+\upsilon X^{(1)}\left(\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\right),\tag{26}$$

将(25)式,(22)式和(19)式代入上式,令等号两边小参数 υ^m 的系数相等,有

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}^{m}+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\left(\frac{\Delta_{j+1,k+1}}{\Delta}R_{k+1}\frac{\partial\xi_{s+1}^{m-1}}{\partial x'_{j+1}}\right)+\frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial}{\partial x'_{l+1}}\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}^{m-1}\right)+\frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial}{\partial x'_{l+1}}\left(\frac{\Delta_{j+1,k+1}}{\Delta}R_{k+1}\frac{\partial\xi_{s+1}^{m-2}}{\partial x'_{j+1}}\right)=X_m^{(1)}(h_{s+1})+X_{m-1}^{(1)}\left(\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\right),\tag{27}$$

其中

$$X_m^{(1)}=\xi_{k+1}^{m}\frac{\partial}{\partial x_{k+1}}+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_{k+1}^{m}\frac{\partial}{\partial x'_{k+1}}+\frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial\xi_{k+1}^{m-1}}{\partial x'_{l+1}}\frac{\partial}{\partial x'_{k+1}},\tag{28}$$

式中 $\xi_{s+1}^{-1}=\xi_{s+1}^{-2}=0$.

对于受到小扰动 υR_α 作用的事件空间中完整力学系统(17),如果无限小生成元 ξ_{s+1}^{m} 满足方程(27),则相应对称性为系统的 Lie 对称性.

定理 1 对于事件空间中受到小扰动 $\upsilon R_\alpha(x_\beta,x'_\beta)$ 作用的完整力学系统,如果生成元 ξ_{s+1}^{m} 满足方程(27),且存在某函数 $\mu=\mu(x_\alpha,x'_\alpha)$ 满足关系

$$\frac{\partial h_{s+1}}{\partial x'_{s+1}}+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\ln\mu=0,\tag{29}$$

$$\frac{\partial}{\partial x'_{s+1}}\left(\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\right)+\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\frac{\partial}{\partial x'_{s+1}}(\ln\mu)=0.\tag{30}$$

则事件空间中完整力学系统存在一类 Hojman 形式的高阶绝热不变量,形如

$$I_z=\upsilon^m\left[\frac{1}{\mu}\frac{\partial}{\partial x_{s+1}}(\mu\xi_{s+1}^{m})+\frac{1}{\mu}\frac{\partial}{\partial x'_{s+1}}\left(\mu\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}^{m}\right)+\frac{1}{\mu}\frac{\partial}{\partial x'_{s+1}}\left(\mu\frac{\Delta_{j+1,k+1}}{\Delta}R_{k+1}\frac{\partial\xi_{s+1}^{m-1}}{\partial x'_{j+1}}\right)\right],\quad(m=0,1,\cdots,z).\tag{31}$$

证明 将 I_z 按(24)式对参数 τ 求导数,有

$$\begin{aligned}\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}I_z&=\upsilon^m\left[\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial\xi_{s+1}^{m}}{\partial x_{s+1}}+\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial}{\partial x'_{s+1}}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}^{m}+\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial}{\partial x'_{s+1}}\left(\frac{\Delta_{j+1,k+1}}{\Delta}R_{k+1}\frac{\partial\xi_{s+1}^{m-1}}{\partial x'_{j+1}}\right)+\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}X_m^{(1)}(\ln\mu)\right]\\&=\upsilon^m\left[\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial\xi_{s+1}^{m}}{\partial x_{s+1}}+\upsilon\frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial^2\xi_{s+1}^{m}}{\partial x'_{l+1}\partial x_{s+1}}+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial}{\partial x'_{s+1}}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}^{m}+\upsilon\frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial^2}{\partial x'_{l+1}\partial x'_{s+1}}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}^{m}\right.\\&\quad+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial}{\partial x'_{s+1}}\left(\frac{\Delta_{j+1,k+1}}{\Delta}R_{k+1}\frac{\partial\xi_{s+1}^{m-1}}{\partial x'_{j+1}}\right)+\upsilon\frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial^2}{\partial x'_{l+1}\partial x'_{s+1}}\left(\frac{\Delta_{j+1,k+1}}{\Delta}R_{k+1}\frac{\partial\xi_{s+1}^{m-1}}{\partial x'_{j+1}}\right)\\&\quad\left.+\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}X_m^{(1)}(\ln\mu)+\upsilon\frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial}{\partial x'_{l+1}}X_m^{(1)}(\ln\mu)\right].\end{aligned}\tag{32}$$

对于任意函数 $\phi(x_\alpha,x'_\alpha)$,有

$$\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\phi=x'_\alpha\frac{\partial\phi}{\partial x_\alpha}+x''_1\frac{\partial\phi}{\partial x'_1}+\left(h_{s+1}+\upsilon\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\right)\frac{\partial\phi}{\partial x'_{s+1}},\tag{33}$$

于是有

$$\begin{aligned}X^{(1)}\left(\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\phi\right)&=x'_\alpha X^{(1)}\left(\frac{\partial\phi}{\partial x_\alpha}\right)+\frac{\partial\phi}{\partial x_{s+1}}\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}+\left(h_{s+1}+\upsilon\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\right)X^{(1)}\left(\frac{\partial\phi}{\partial x'_{s+1}}\right)\\&\quad+x''_1X^{(1)}\left(\frac{\partial\phi}{\partial x'_1}\right)+\frac{\partial\phi}{\partial x'_{s+1}}\left[X^{(1)}(h_{s+1})+\upsilon X^{(1)}\left(\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\right)\right],\end{aligned}\tag{34}$$

而

$$\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}X^{(1)}(\phi)=\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}\frac{\partial\phi}{\partial x_{s+1}}+\xi_{s+1}\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial\phi}{\partial x_{s+1}}+\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}\frac{\partial\phi}{\partial x'_{s+1}}+\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial\phi}{\partial x'_{s+1}},\tag{35}$$

容易验证

$$\xi_{s+1}\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial\phi}{\partial x_{s+1}}+\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\xi_{s+1}\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\frac{\partial\phi}{\partial x'_{s+1}}=x'_\alpha X^{(1)}\left(\frac{\partial\phi}{\partial x_\alpha}\right)+x''_1X^{(1)}\left(\frac{\partial\phi}{\partial x'_1}\right)+\left(h_{s+1}+\upsilon\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\right)X^{(1)}\left(\frac{\partial\phi}{\partial x'_{s+1}}\right),\tag{36}$$

因此,由(34)—(36)式,如果无限小生成元 ξ_{s+1} 满足 Lie 对称性确定方程(26),则下述关系成立:

$$X^{(1)}\left(\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}\phi\right)=\frac{\tilde{\mathrm{d}}}{\mathrm{d}\tau}X^{(1)}(\phi).\tag{37}$$

将(25)式,(22)式代入(37)式,令等号两边 υ^m 的系数相等,并注意到(28)式,有

$$\frac{\tilde{d}}{d\tau}X_m^{(1)}(\phi) + \frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\frac{\partial}{\partial x'_{s+1}}X_{m-1}^{(1)}(\phi) = X_m^{(1)}\left(\frac{\tilde{d}}{d\tau}\phi\right) + X_{m-1}^{(1)}\left(\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\frac{\partial\phi}{\partial x'_{s+1}}\right). \quad (38)$$

将确定方程(26)对 x'_{s+1} 求偏导数，有

$$\frac{\partial}{\partial x'_{s+1}}\left[\frac{\tilde{d}}{d\tau}\frac{\tilde{d}}{d\tau}\xi_{s+1} - X^{(1)}(h_{s+1}) - \upsilon X^{(1)}\left(\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\right)\right]$$
$$= \frac{\tilde{d}}{d\tau}\frac{\partial\xi_{s+1}}{\partial x_{s+1}} + \frac{\tilde{d}}{d\tau}\frac{\partial}{\partial x'_{s+1}}\frac{\tilde{d}}{d\tau}\xi_{s+1} - X^{(1)}\left(\frac{\partial h_{s+1}}{\partial x'_{s+1}}\right) - \upsilon X^{(1)}\left[\frac{\partial}{\partial x'_{s+1}}\left(\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\right)\right] = 0, \quad (39)$$

将(25)式，(22)式代入(39)式，令等号两边 υ^m 的系数相等，得到

$$\frac{\bar{d}}{d\tau}\frac{\partial\xi_{s+1}^m}{\partial x_{s+1}} + \frac{\bar{d}}{d\tau}\frac{\partial}{\partial x'_{s+1}}\frac{\bar{d}}{d\tau}\xi_{s+1}^m + \frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial^2\xi_{s+1}^{m-1}}{\partial x'_{l+1}\partial x_{s+1}} + \frac{\bar{d}}{d\tau}\frac{\partial}{\partial x'_{s+1}}\left(\frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial\xi_{s+1}^{m-1}}{\partial x'_{l+1}}\right)$$
$$+ \frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial^2}{\partial x'_{s+1}\partial x'_{l+1}}\left(\frac{\bar{d}}{d\tau}\xi_{s+1}^{m-1}\right) + \frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial^2}{\partial x'_{s+1}\partial x'_{l+1}}\left(\frac{\Delta_{j+1,k+1}}{\Delta}R_{k+1}\frac{\partial\xi_{s+1}^{m-2}}{\partial x'_{j+1}}\right)$$
$$- X_m^{(1)}\left(\frac{\partial h_{s+1}}{\partial x'_{s+1}}\right) - X_{m-1}^{(1)}\left[\frac{\partial}{\partial x'_{s+1}}\left(\frac{\Delta_{s+1,k+1}}{\Delta}R_{k+1}\right)\right] = 0. \quad (40)$$

将(40)式代入(32)式，利用条件(29)式和(30)式，并注意关系(38)式，有

$$\frac{\tilde{d}}{d\tau}I_z = \upsilon^m\frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial}{\partial x'_{l+1}}\left\{\upsilon X_m^{(1)}(\ln\mu) - X_{m-1}^{(1)}(\ln\mu) + \frac{\partial}{\partial x_{s+1}}(\upsilon\xi_{s+1}^m - \xi_{s+1}^{m-1}) + \frac{\partial}{\partial x'_{s+1}}\left(\upsilon\frac{\bar{d}}{d\tau}\xi_{s+1}^m - \frac{\bar{d}}{d\tau}\xi_{s+1}^{m-1}\right) + \frac{\partial}{\partial x'_{s+1}}\left[\frac{\Delta_{j+1,k+1}}{\Delta}R_{k+1}\frac{\partial}{\partial x'_{j+1}}(\upsilon\xi_{s+1}^{m-1} - \xi_{s+1}^{m-2})\right]\right\}$$
$$= \upsilon^{z+1}\frac{\Delta_{l+1,r+1}}{\Delta}R_{r+1}\frac{\partial}{\partial x'_{l+1}}\left[X_z^{(1)}(\ln\mu) + \frac{\partial\xi_{s+1}^z}{\partial x_{s+1}} + \frac{\partial}{\partial x'_{s+1}}\left(\frac{\bar{d}}{d\tau}\xi_{s+1}^z + \frac{\Delta_{j+1,k+1}}{\Delta}R_{k+1}\frac{\partial}{\partial x'_{j+1}}\xi_{s+1}^{z-1}\right)\right]. \quad (41)$$

上式表明 $\frac{\tilde{d}}{d\tau}I_z$ 与 υ^{z+1} 成正比，由定义 1，I_z 是事件空间中完整系统的一个 z 阶绝热不变量.证毕.

4.算　　例

例　事件空间中完整系统为[28]

$$\Lambda = x'_1\left\{\frac{1}{2}\left[\left(\frac{x'_2}{x'_1}\right)^2 + \left(\frac{x'_3}{x'_1}\right)^2\right] - x_3\right\}, \quad (42)$$

$$P''_1 = 0, P''_2 = -x'_3, P''_3 = x'_2, \quad (43)$$

试研究系统 Lie 对称性的摄动与 Hojman 绝热不变量.

方程(7)给出

$$x''_2 = -x'_1x'_3 + \frac{x'_2}{x'_1}x''_1,$$
$$x''_3 = -(x'_1)^2 + x'_1x'_2 + \frac{x'_3}{x'_1}x''_1, \quad (44)$$

确定方程(13)给出

$$\frac{\bar{d}}{d\tau}\frac{\bar{d}}{d\tau}\xi_2^0 = \frac{x''_1}{x'_1}\frac{\bar{d}}{d\tau}\xi_2^0 - x'_1\frac{\bar{d}}{d\tau}\xi_3^0,$$
$$\frac{\bar{d}}{d\tau}\frac{\bar{d}}{d\tau}\xi_3^0 = x'_1\frac{\bar{d}}{d\tau}\xi_2^0 + \frac{x''_1}{x'_1}\frac{\bar{d}}{d\tau}\xi_3^0, \quad (45)$$

它有解

$$\xi_2^0 = 1, \xi_3^0 = 0, \quad (46)$$

(14)式给出

$$2\frac{x''_1}{x'_1} + \frac{\bar{d}}{d\tau}\ln\mu_0 = 0, \quad (47)$$

方程(47)有解

$$\mu_0 = (x'_1)^{-2}\left(\frac{x'_3}{x'_1} - x_2 + x_1\right). \quad (48)$$

根据定理 1，由(46)式和(48)式，求得此未受扰系统的 Lie 对称性导致的 Hojman 守恒量为

$$I_0 = -\left(\frac{x'_3}{x'_1} - x_2 + x_1\right)^{-1} = \text{const}. \quad (49)$$

下面我们来求系统的一阶绝热不变量.假设系统受到的小扰动为

$$\upsilon R_1 = \upsilon x'_1\left[\left(\frac{x'_2}{x'_1}\right)^2 - \left(\frac{x'_3}{x'_1}\right)^2\right],$$
$$\upsilon R_2 = -\upsilon x'_2, \upsilon R_3 = \upsilon x'_3, \quad (50)$$

方程(18)给出

$$x''_2 = -x'_1x'_3 + \frac{x'_2}{x'_1}x''_1 - \upsilon x'_1x'_2,$$
$$x''_3 = -(x'_1)^2 + x'_1x'_2 + \frac{x'_3}{x'_1}x''_1 + \upsilon x'_1x'_3, \quad (51)$$

当 $m = 1$ 时，方程(27)给出

$$\frac{\bar{d}}{d\tau}\frac{\bar{d}}{d\tau}\xi_2^1 + \frac{\bar{d}}{d\tau}\left(-x'_1x'_2\frac{\partial\xi_2^0}{\partial x'_2} + x'_1x'_3\frac{\partial\xi_2^0}{\partial x'_3}\right) - x'_1x'_2\frac{\partial}{\partial x'_2}\left(\frac{\bar{d}}{d\tau}\xi_2^0\right) + x'_1x'_3\frac{\partial}{\partial x'_3}\left(\frac{\bar{d}}{d\tau}\xi_2^0\right)$$

$$
\begin{aligned}
&= \frac{x_1''}{x_1'}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_2^1 - x_1'\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_3^1 - x_1''x_2'\frac{\partial \xi_2^0}{\partial x_2'} + (x_1')^2 x_2'\frac{\partial \xi_3^0}{\partial x_2'} \\
&\quad + x_1''x_3'\frac{\partial \xi_2^0}{\partial x_3'} - (x_1')^2 x_3'\frac{\partial \xi_3^0}{\partial x_3'} - x_1'\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_2^0, \\
&\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_3^1 + \frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\left(-x_1'x_2'\frac{\partial \xi_3^0}{\partial x_2'} + x_1'x_3'\frac{\partial \xi_3^0}{\partial x_3'}\right) \\
&\quad - x_1'x_2'\frac{\partial}{\partial x_2'}\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_3^0\right) + x_1'x_3'\frac{\partial}{\partial x_3'}\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_3^0\right) \\
&= x_1'\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_2^1 + \frac{x_1''}{x_1'}\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_3^1 - (x_1')^2 x_2'\frac{\partial \xi_2^0}{\partial x_2'} \\
&\quad - x_1''x_2'\frac{\partial \xi_3^0}{\partial x_2'} + (x_1')^2 x_3'\frac{\partial \xi_2^0}{\partial x_3'} + x_1''x_3'\frac{\partial \xi_3^0}{\partial x_3'} \\
&\quad + x_1'\frac{\tilde{d}}{\mathrm{d}\tau}\xi_3^0, \qquad (52)
\end{aligned}
$$

方程(52)有解

$$
\xi_2^1 = 0, \xi_3^1 = \left(\frac{x_2'}{x_1'} + x_3\right)^2, \qquad (53)
$$

(29)式,(30)式给出

$$
2\frac{x_1''}{x_1'} + \frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\ln\mu = 0, \qquad (54)
$$

$$
-x_1'x_2'\frac{\partial}{\partial x_2'}(\ln\mu) + x_1'x_3'\frac{\partial}{\partial x_3'}(\ln\mu) = 0, \qquad (55)
$$

方程(54),(55)有解

$$
\mu = (x_1')^{-2}, \qquad (56)
$$

由(53)式,(56)式,根据定理2,系统存在一阶Hojman绝热不变量,形如

$$
I_1 = -\left(\frac{x_3'}{x_1'} - x_2 + x_1\right)^{-1} + 2\upsilon\left(\frac{x_2'}{x_1'} + x_3\right), \qquad (57)
$$

进一步可求得系统的更高阶绝热不变量.

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Lie symmetries and adiabatic invariants for holonomic systems in event space*

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（Received 23 August 2006；revised manuscript received 14 October 2006）

Abstract

The perturbations of Lie symmetries and adiabatic invariants for holonomic mechanical systems in event space are studied. Based on the definition of high-order adiabatic invariants of a mechanical system，the perturbation of Lie symmetries for the system under the action of small disturbance is investigated，and a type of Hojman high-order adiabatic invariants of the system are obtained. The conditions for the existence of the adiabatic invariants and the form of the adiabatic invariants are given. An example is presented to illustrate the results.

Keywords：event space，symmetry，perturbation，adiabatic invariant

PACC：0320

* Project supported by the Natural Science Foundation of High Education of Jiangsu Province of China（Grant No. 04KJA130135）.

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