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电各向异性色散介质电磁散射的三维递推卷积-时域有限差分方法分析*

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根据递推卷积原理，将磁化等离子体的频域介电系数过渡到时域，通过引入时域复数极化率张量和时域复数电位移矢量，得到了磁化等离子体的三维时域有限差分方法迭代式. 为了验证该方法，用它计算了非磁等离子体球的后向雷达散射截面，与移位算子法结果符合很好. 应用该方法计算和分析了磁化等离子体球的电磁波散射，发现其后向散射时域波形明显出现了交叉极化分量.

关键词：递推卷积，磁化等离子体，电磁散射，时域有限差分方法

PACC：4110H，5170

1. 引 言

等离子体是一种色散介质，对其电磁性质的研究在近 10 年来一直都是热门课题之一，并且在研究方法上出现了大量处理色散介质电磁仿真的方法，尤其是时域有限差分(FDTD)方法，如递推卷积(RC)法[1]、辅助方程法[2]、Z 变换法[3]、电流密度卷积法[4]、分段线性递推卷积法[5]和移位算子法[6,7]、交替方向隐式 FDTD 方法[8]. 在外加磁场情况下，像等离子体和铁氧体分别呈现出电各向异性和磁各向异性. 对于磁化铁氧体材料，文献[9,10]分别研究了自旋电流及自旋极化电流的影响和作用，作者也在最近提出了一种基于矩阵 Padé 逼近理论的三维 FDTD(Padé-FDTD)方法，并分析了磁各向异性介质的电磁散射[11]，然而，由于处理电各向异性色散介质的复杂性，电各向异性色散介质的 FDTD 方法研究尚不很多，主要有直接积分法[12]、辅助方程法[13]和推广的 RC 法[14]. 这些方法只限于分析一维问题，对三维问题还没有进行讨论. 本文实现的是处理电各向异性色散介质的三维 RC-FDTD 方法，并分析了磁化等离子体球的电磁散射. 算例表明本文方法正确有效.

2. 磁化等离子体磁化系数的时域表达式

假设外加磁场平行于 z 轴，磁等离子体的复介电系数张量可用磁化系数张量 $\chi(\omega)$ 描述如下：

$$\begin{aligned}\hat{\boldsymbol{\varepsilon}}_{ij}(\omega) &= \varepsilon_0(1+\boldsymbol{\chi}(\omega)) \\ &= \varepsilon_0\begin{bmatrix} 1+\chi_{xx}(\omega) & \mathrm{j}\chi_{xy}(\omega) & 0 \\ -\mathrm{j}\chi_{yx}(\omega) & 1+\chi_{yy}(\omega) & 0 \\ 0 & 0 & 1+\chi_{zz}(\omega)\end{bmatrix},\end{aligned} \tag{1}$$

式中 $\chi_{ij}(i,j=x,y,z)$ 为磁化系数张量的分量. 对 Drude 磁等离子体，介电常数 $\chi_{ij}(i,j=x,y,z)$ 的非零分量分别为

$$\begin{aligned}\chi_{xx}(\omega) &= \chi_{yy}(\omega) \\ &= -\frac{(\omega_{\mathrm{p}}/\omega)^2(1-\mathrm{j}\nu_{\mathrm{c}}/\omega)}{(1-\mathrm{j}\nu_{\mathrm{c}}/\omega)^2-(\omega_{\mathrm{b}}/\omega)^2},\end{aligned} \tag{2}$$

$$\begin{aligned}\chi_{xy}(\omega) &= \chi_{yx}(\omega) \\ &= \frac{(\omega_{\mathrm{p}}/\omega)^2(\omega_{\mathrm{b}}/\omega)}{(1-\mathrm{j}\nu_{\mathrm{c}}/\omega)^2-(\omega_{\mathrm{b}}/\omega)^2},\end{aligned} \tag{3}$$

$$\chi_{zz}(\omega) = -\frac{\omega_{\mathrm{p}}^2}{\omega(\omega+\mathrm{j}\nu_{\mathrm{c}})}. \tag{4}$$

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χ_{zz} 与各向同性等离子体有相同表达式[1].

将(2)—(4)式分别做逆傅里叶变换,得 $\chi_{ij}(i,j=x,y,z)$ 时域表达式

$$\chi_{xx}(t)=\chi_{yy}(t)=\frac{\omega_p^2}{\nu_c^2+\omega_b^2}(\nu_c-e^{-\nu_c t}[\nu_c\cos(\omega_b t)-\omega_b\sin(\omega_b t)])U(t), \tag{5}$$

$$\chi_{xy}(t)=\chi_{yx}(t)=\frac{\omega_p^2}{\nu_c^2+\omega_b^2}(\omega_b-e^{-\nu_c t}[\omega_b\cos(\omega_b t)+\nu_c\sin(\omega_b t)])U(t), \tag{6}$$

$$\chi_{zz}(t)=\frac{\omega_p^2}{\nu_c}(1-e^{-\nu_c t})U(t). \tag{7}$$

为了便于计算下面用到的指数函数的卷积积分,将(5),(6)式写成如下的时域复数磁化系数:

$$\hat{\chi}_{xx}(t)=\hat{\chi}_{yy}(t)=\frac{\omega_p^2}{\nu_c^2+\omega_b^2}(\nu_c+j\omega_b)\times(1-e^{-(\nu_c-j\omega_b)t})U(t), \tag{8}$$

$$\hat{\chi}_{xy}(t)=\hat{\chi}_{yx}(t)=\frac{\omega_p^2}{\nu_c^2+\omega_b^2}(\omega_b-j\nu_c)\times(1-e^{-(\nu_c-j\omega_b)t})U(t), \tag{9}$$

即

$$\begin{aligned}\chi_{xx}(t)&=\mathrm{Re}[\hat{\chi}_{xx}(t)],\\ \chi_{xy}(t)&=\mathrm{Re}[\hat{\chi}_{xy}(t)].\end{aligned} \tag{10}$$

3. RC-FDTD 算法原理分析

麦克斯韦旋度方程为

$$\nabla\times\boldsymbol{H}=\frac{\partial\boldsymbol{D}}{\partial t}, \tag{11}$$

$$\nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{H}}{\partial t}. \tag{12}$$

$\boldsymbol{D}(\omega)$ 和 $\boldsymbol{E}(\omega)$ 之间本构关系式为

$$\boldsymbol{D}(\omega)=\hat{\boldsymbol{\varepsilon}}_{ij}(\omega)\cdot\boldsymbol{E}. \tag{13}$$

将(1)式代入(13)式得

$$D_x(\omega)=\varepsilon_0(1+\chi_{xx}(\omega))E_x(\omega)+j\chi_{xy}(\omega)\varepsilon_0E_y(\omega), \tag{14}$$

$$D_y(\omega)=\varepsilon_0(1+\chi_{yy}(\omega))E_y(\omega)-j\chi_{yx}(\omega)\varepsilon_0E_x(\omega), \tag{15}$$

$$D_z(\omega)=\varepsilon_0(1+\chi_{zz}(\omega))E_z(\omega). \tag{16}$$

在时域,(14),(15)和(16)式分别为卷积关系,即

$$\frac{D_x(t)}{\varepsilon_0}=E_x(t)+\chi_{xx}(t)*E_x(t)+\chi_{xy}(t)*E_y(t), \tag{17}$$

$$\frac{D_y(t)}{\varepsilon_0}=E_y(t)+\chi_{yy}(t)*E_y(t)-\chi_{yx}(t)*E_x(t), \tag{18}$$

$$\frac{D_z(t)}{\varepsilon_0}=E_z(t)+\chi_{zz}(t)*E_z(t). \tag{19}$$

根据卷积积分,将(17)—(18)式的卷积积分在 n 时刻进行离散,并运用复磁化系数(7)—(9)式,得

$$\frac{\hat{D}_x^n}{\varepsilon_0}=E_x^n+\int_0^{n\Delta t}E_x(n\Delta t-\tau)\hat{\chi}_{xx}(\tau)d\tau+\int_0^{n\Delta t}E_y(n\Delta t-\tau)\hat{\chi}_{xy}(\tau)d\tau, \tag{20}$$

$$\frac{\hat{D}_y^n}{\varepsilon_0}=E_y^n+\int_0^{n\Delta t}E_y(n\Delta t-\tau)\chi_{yy}(\tau)d\tau-\int_0^{n\Delta t}E_x(n\Delta t-\tau)\chi_{yx}(\tau)d\tau, \tag{21}$$

$$\frac{\hat{D}_z^n}{\varepsilon_0}=E_z^n+\int_0^{n\Delta t}E_z(n\Delta t-\tau)\chi_{zz}(\tau)d\tau. \tag{22}$$

应该注意的是,在电磁的迭代更新过程中只用到复电位移矢量的实部.

在 FDTD 迭代计算过程中,每个时间步 Δt 内电场的三个分量可以近似看作是常量,并把卷积积分写成卷积和的形式,因此(20)式可变为

$$\frac{\hat{D}_x^n}{\varepsilon_0}=E_x^n+\sum_{m=0}^{n-1}E_x^{n-m}\int_{m\Delta t}^{(m+1)\Delta t}\hat{\chi}_{xx}(\tau)d\tau+\sum_{m=0}^{n-1}E_y^{n-m}\int_{m\Delta t}^{(m+1)\Delta t}\hat{\chi}_{xy}(\tau)d\tau. \tag{23}$$

于是

$$\begin{aligned}\frac{\hat{D}_x^{n+1}-\hat{D}_x^n}{\varepsilon_0}=&(1+\chi_{xx}^0)E_x^{n+1}-E_x^n+\chi_{xy}^0E_y^{n+1}\\&+\sum_{m=0}^{n-1}E_x^{n-m}\Big[\int_{(m+1)\Delta t}^{(m+2)\Delta t}\hat{\chi}_{xx}(\tau)d\tau-\int_{m\Delta t}^{(m+1)\Delta t}\hat{\chi}_{xx}(\tau)d\tau\Big]\\&+\sum_{m=0}^{n-1}E_y^{n-m}\Big[\int_{(m+1)\Delta t}^{(m+2)\Delta t}\hat{\chi}_{xy}(\tau)d\tau-\int_{m\Delta t}^{(m+1)\Delta t}\hat{\chi}_{xy}(\tau)d\tau\Big],\end{aligned} \tag{24}$$

式中

$$\chi^0_{xx}(t) = \mathrm{Re}\left[\int_0^{\Delta t}\widehat{\chi}_{xx}(\tau)\,\mathrm{d}\tau\right],\quad \chi^0_{xy}(t) = \mathrm{Re}\left[\int_0^{\Delta t}\widehat{\chi}_{xy}(\tau)\,\mathrm{d}\tau\right],\tag{25}$$

$$\chi^m_{xx} = \mathrm{Re}\left[\int_{m\Delta t}^{(m+1)\Delta t}\widehat{\chi}_{xx}(\tau)\,\mathrm{d}\tau\right],\quad \Delta\chi^m_{xx} = \chi^m_{xx} - \chi^{m+1}_{xx},\tag{26}$$

$$\chi^m_{xy} = \mathrm{Re}\left[\int_{m\Delta t}^{(m+1)\Delta t}\widehat{\chi}_{xy}(\tau)\,\mathrm{d}\tau\right],\quad \Delta\chi^m_{xy} = \chi^m_{xy} - \chi^{m+1}_{xy}.\tag{27}$$

由(24)和(11)式得到

$$E_x^{n+1} = (1+\chi^0_{xx})^{-1}\Big[E_x^n - \chi^0_{xy}E_y^{n+1} + \sum_{m=0}^{n-1}E_x^{n-m}\Delta\chi^m_{xx} + \sum_{m=0}^{n-1}E_y^{n-m}\Delta\chi^m_{xy} + \frac{\Delta t}{\varepsilon_0}(\nabla\times \boldsymbol{H})_x^{n+1/2}\Big].\tag{28}$$

同理可得

$$E_y^{n+1} = (1+\chi^0_{yy})^{-1}\Big[E_y^n + \chi^0_{xy}E_x^{n+1} + \sum_{m=0}^{n-1}E_y^{n-m}\Delta\chi^m_{yy} - \sum_{m=0}^{n-1}E_x^{n-m}\Delta\chi^m_{yx} + \frac{\Delta t}{\varepsilon_0}(\nabla\times \boldsymbol{H})_y^{n+1/2}\Big].\tag{29}$$

$$E_z^{n+1} = (1+\chi^0_{zz})^{-1}\Big[E_z^n + \sum_{m=0}^{n-1}E_z^{n-m}\Delta\chi^m_{zz} + \frac{\Delta t}{\varepsilon_0}(\nabla\times \boldsymbol{H})_z^{n+1/2}\Big],\tag{30}$$

$$\chi^0_{yy} = \mathrm{Re}\left[\int_0^{\Delta t}\widehat{\chi}_{yy}(\tau)\,\mathrm{d}\tau\right],\tag{31}$$

$$\chi^0_{xy} = \chi^0_{yx} = \mathrm{Re}\left[\int_0^{\Delta t}\widehat{\chi}_{yx}(\tau)\,\mathrm{d}\tau\right],$$

$$\int_{m\Delta t}^{(m+1)\Delta t}\widehat{\chi}_{yy}(\tau)\,\mathrm{d}\tau = \widehat{\chi}^m_{yy} = \widehat{\chi}^m_{xx},\quad \Delta\widehat{\chi}^m_{yy} = \widehat{\chi}^m_{yy} - \widehat{\chi}^{m+1}_{yy},\tag{32}$$

$$\int_{m\Delta t}^{(m+1)\Delta t}\widehat{\chi}_{yx}(\tau)\,\mathrm{d}\tau = \widehat{\chi}^m_{yx} = \widehat{\chi}^m_{xy},\quad \Delta\widehat{\chi}^m_{yx} = \widehat{\chi}^m_{yx} - \widehat{\chi}^{m+1}_{yx},\tag{33}$$

$$\chi^0_{zz} = \int_0^{\Delta t}\chi_{zz}(\tau)\,\mathrm{d}\tau,\quad \chi^m_{zz} = \int_{m\Delta t}^{(m+1)\Delta t}\chi_{zz}(\tau)\,\mathrm{d}\tau,\quad \Delta\chi^m_{zz} = \chi^m_{zz} - \chi^{m+1}_{zz}.\tag{34}$$

由(28),(29)式知，E_x 和 E_y 相互耦合，所以必须联立求解．根据文献[1,14]，通过引入中间辅助变量 $\widehat{\psi}^n_{xx}$，$\widehat{\psi}^n_{yxy}$，$\widehat{\psi}^n_{yy}$，$\widehat{\psi}^n_{xyx}$，$\widehat{\psi}^n_{zz}$，设

$$\widehat{\psi}^n_{xx} = \sum_{m=0}^{n-1}E_x^{n-m}\Delta\widehat{\chi}^m_{xx},$$

$$\widehat{\psi}^n_{yxy} = \sum_{m=0}^{n}E_y^{n-m}\Delta\widehat{\chi}^m_{xy},$$

$$\widehat{\psi}^n_{yy} = \sum_{m=0}^{n}E_y^{n-m}\Delta\widehat{\chi}^m_{yy},$$

$$\widehat{\psi}^n_{xyx} = \sum_{m=0}^{n}E_x^{n-m}\Delta\widehat{\chi}^m_{yx},$$

$$\widehat{\psi}^n_{zz} = \sum_{m=0}^{n}E_z^{n-m}\Delta\widehat{\chi}^m_{zz},$$

并令

$$\psi^n_{xx} = \mathrm{Re}(\widehat{\psi}^n_{xx}),\quad \psi^n_{yxy} = \mathrm{Re}(\widehat{\psi}^n_{yxy}),\quad \psi^n_{yy} = \mathrm{Re}(\widehat{\psi}^n_{yy}),\quad \psi^n_{xyx} = \mathrm{Re}(\widehat{\psi}^n_{xyx}),$$

改写(28),(29)式可得

$$E_x^{n+1} = C_AE_x^n + C_B(\nabla\times\boldsymbol{H})_x + C_C(\psi^n_{xx}+\psi^n_{yxy}) - C_DE_y^n - C_E(\nabla\times\boldsymbol{H})_y - C_F(\psi^n_{yy}-\psi^n_{xyx}),\tag{35}$$

$$E_y^{n+1} = C_AE_y^n + C_B(\nabla\times\boldsymbol{H})_y + C_C(\psi^n_{yy}-\psi^n_{xyx}) + C_DE_x^n + C_E(\nabla\times\boldsymbol{H})_x + C_F(\psi^n_{xx}-\psi^n_{yxy}),\tag{36}$$

$$E_z^{n+1} = C_GE_x^n + C_H(\nabla\times\boldsymbol{H})_z + C_I\psi^n_{zz},\tag{37}$$

式中

$$C_A = \frac{\left(1+\chi^0_{yy}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)\left(1-\frac{\sigma\Delta t}{2\varepsilon_0}\right)}{\left(1+\chi^0_{xx}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)\left(1+\chi^0_{yy}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)+(\chi^0_{xy})^2},$$

$$C_B = \frac{\left(1+\chi^0_{yy}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)}{\left(1+\chi^0_{xx}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)\left(1+\chi^0_{yy}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)+(\chi^0_{xy})^2}\frac{\Delta t}{\varepsilon_0},$$

$$C_C = \frac{\left(1+\chi^0_{yy}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)}{\left(1+\chi^0_{xx}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)\left(1+\chi^0_{yy}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)+(\chi^0_{xy})^2},$$

$$C_D = \frac{\chi^0_{xy}\left(1-\frac{\sigma\Delta t}{2\varepsilon_0}\right)}{\left(1+\chi^0_{xx}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)\left(1+\chi^0_{yy}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)+(\chi^0_{xy})^2},$$

$$C_E = \frac{\chi^0_{xy}}{\left(1+\chi^0_{xx}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)\left(1+\chi^0_{yy}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)+(\chi^0_{xy})^2}\frac{\Delta t}{\varepsilon_0},$$

$$C_F = \frac{\chi^0_{xy}}{\left(1+\chi^0_{xx}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)\left(1+\chi^0_{yy}+\frac{\sigma\Delta t}{2\varepsilon_0}\right)+(\chi^0_{yy})^2},$$

$$C_G = \frac{1 - \dfrac{\sigma\Delta t}{2\varepsilon_0}}{1 + \chi_{zz}^0 + \dfrac{\sigma\Delta t}{2\varepsilon_0}},$$

$$C_H = \frac{1}{\left(1 + \chi_{zz}^0 + \dfrac{\sigma\Delta t}{2\varepsilon_0}\right)}\frac{\Delta t}{\varepsilon_0},$$

$$C_I = \frac{1}{\left(1 + \chi_{zz}^0 + \dfrac{\sigma\Delta t}{2\varepsilon_0}\right)}.$$

应该注意的是，在 FDTD 迭代计算中只用到复数辅助变量 $\widehat{\psi}_{xx}^n$，$\widehat{\psi}_{yxy}^n$，$\widehat{\psi}_{yy}^n$，$\widehat{\psi}_{xyx}^n$ 的实部. 中间辅助变量 $\widehat{\psi}_{xx}^n$，$\widehat{\psi}_{yxy}^n$，$\widehat{\psi}_{yy}^n$，$\widehat{\psi}_{xyx}^n$，$\widehat{\psi}_{zz}^n$ 的计算见附录. (35)—(37)式中的磁场都在半整数时间步.

以上是电场三个分量的 FDTD 迭代式，至于磁场分量的迭代式与常规 FDTD 相同. 另外，对某一空间节点进行空间离散时，若某些场量不在离散节点位置，需要将相邻节点的场量进行空间插值过渡到该节点，如在计算 $E_x^{n+1}(i+1/2, j, k)$ 时，$E_y^n(i+1/2, j, k)$ 需进行如下的处理：

$$E_y\Big|_{i+1/2,j,k}^{n} = \frac{1}{4}\Big[E_y\Big|_{i,j+1/2,k}^{n} + E_y\Big|_{i,j-1/2,k}^{n} + E_y\Big|_{i+1,j+1/2,k}^{n} + E_y\Big|_{i+1,j-1/2,k}^{n}\Big]. \quad (38)$$

其他场分量的处理与此类同.

4. 算例验证及数值结果

4.1. 算例验证

计算非磁化等离子体球的后向雷达散射截面(RCS). 球的半径为 1.0 m，等离子体电子回旋频率 $\omega_{sb} = 0$，等离子体频率 $\omega_p = 2\pi \times 28.7 \times 10^9$ rad/s，等离子碰撞频率 $\nu_c = 2.0 \times 10^{10}$ Hz. FDTD 计算中 $\delta = 5$ cm，$\Delta t = \delta/(2c)$，c 为光速，入射波为高斯脉冲

$$E_i(t) = \exp[-4\pi(t - t_0)^2/\tau^2], \quad (39)$$

沿着 z 轴入射，其中 $\tau = 60\Delta t$ 和 $t_0 = 0.8\tau$. 计算结果如图 1 所示，其中实线为本文计算结果，圆圈表示用移位算子法[6,7]的计算结果，两结果符合很好.

4.2. 数值结果

当外加磁场平行于 z 轴时，球的半径为 1.0 m，等离子体电子回旋频率 $\omega_b = 3.0 \times 10^{11}$ rad/s，等离子体频率 $\omega_p = 2\pi \times 28.7 \times 10^9$ rad/s，等离子碰撞频率 $\nu_c = 2.0 \times 10^{10}$ Hz. 当入射波沿着 z 轴正向入射，平行于 x 轴极化，则该磁化等离子体球的同极化后向接收的时域波形如图 2(a) 所示，同时也出现后向散射的交叉极化分量，其时域波形如图 2(b) 所示. 作为比较，图 2(b) 给出了在没有外加磁场时等离子体球的后向交叉极化时域波形. 由于此时等离子体球为各向同性介质，后向散射中没有交叉极化分量.

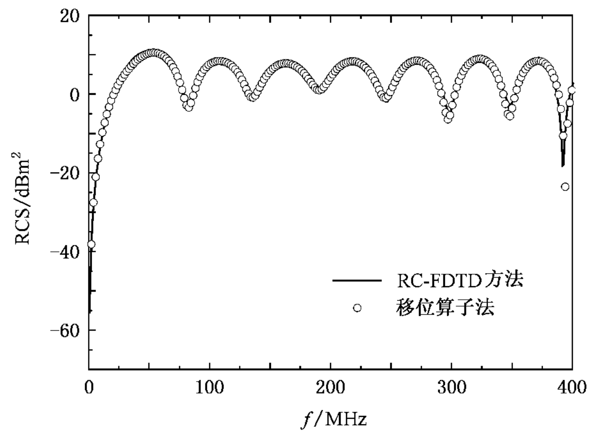

图 1 等离子体球的后向 RCS

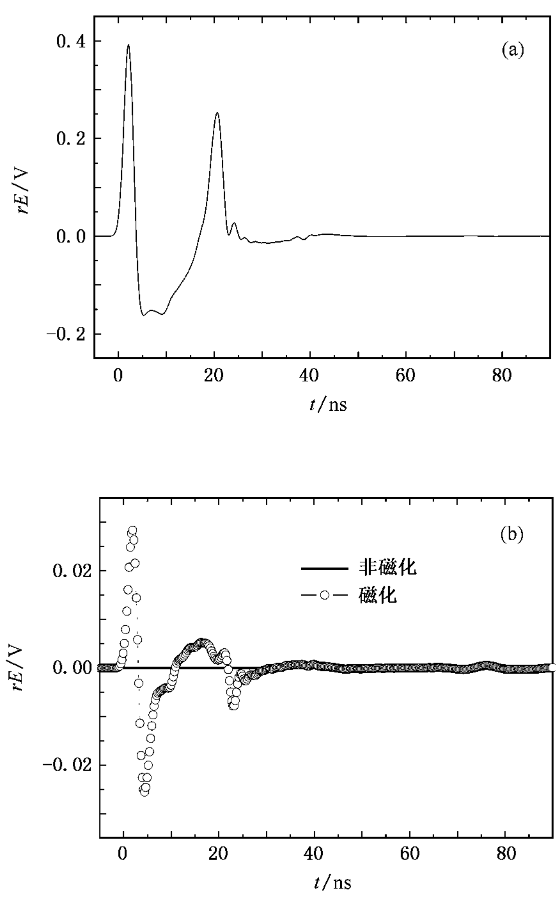

图 2 磁化等离子体球的后向散射时域波形 (a) 同极化，(b) 交叉极化

5. 结 论

研究了具有电各向异性色散特性的磁化等离子体介质的三维 RC-FDTD 方法，并用该方法计算了非磁等离子体球的后向 RCS，与移位算子法结果符合很好. 数值结果表明，该方法是正确有效的. 另外，用该方法计算并分析了磁化等离子体球对电磁波散射的极化特性，发现当等离子体球在外加磁场作用下，后向散射时域波形明显出现了交叉极化分量.

附录 方程(35)—(37)中的 $\widehat{\psi}^{n}_{xx}$，$\widehat{\psi}^{n}_{yxy}$，$\widehat{\psi}^{n}_{yy}$，$\widehat{\psi}^{n}_{xyx}$，$\widehat{\psi}^{n}_{zz}$ 的计算

$\widehat{\psi}^{n}_{xx}$，$\widehat{\psi}^{n}_{yxy}$，$\widehat{\psi}^{n}_{yy}$，$\widehat{\psi}^{n}_{xyx}$，$\widehat{\psi}^{n}_{zz}$ 的计算表达式可用归纳法得到.

$$\widehat{\psi}^{n}_{xx} = \sum_{m=0}^{n-1} E_x^{n-m} \Delta \widehat{\chi}^{m}_{xx}. \tag{A1}$$

下面计算 $\widehat{\psi}^{n}_{xx}$ 的前三个时间步. 当 $n=1$ 时，

$$\widehat{\psi}^{1}_{xx} = \sum_{m=0}^{0} E_x^{1-m} \Delta \widehat{\chi}^{m}_{xx} = E_x^1 \Delta \widehat{\chi}^{0}_{xx}. \tag{A2}$$

当 $n=2$ 时，

$$\widehat{\psi}^{2}_{xx} = \sum_{m=0}^{1} E_x^{2-m} \Delta \widehat{\chi}^{m}_{xx} = E_x^2 \Delta \widehat{\chi}^{0}_{xx} + E_x^1 \Delta \widehat{\chi}^{1}_{xx}. \tag{A3}$$

当 $n=3$ 时，

$$\begin{aligned}\widehat{\psi}^{3}_{xx} &= \sum_{m=0}^{2} E_x^{3-m} \Delta \widehat{\chi}^{m}_{xx} \\ &= E_x^3 \Delta \widehat{\chi}^{0}_{xx} + E_x^2 \Delta \widehat{\chi}^{1}_{xx} + E_x^1 \Delta \widehat{\chi}^{2}_{xx}.\end{aligned} \tag{A4}$$

由(26)式知，

$$\Delta \widehat{\chi}^{m+1}_{xx} = \mathrm{e}^{-(\nu_c - \mathrm{j}\omega_b)\Delta t} \Delta \widehat{\chi}^{m}_{xx}. \tag{A5}$$

于是

$$\begin{aligned}\widehat{\psi}^{2}_{xx} &= E_x^2 \Delta \widehat{\chi}^{0}_{xx} + E_x^1 \Delta \widehat{\chi}^{0}_{xx} \mathrm{e}^{-(\nu_c - \mathrm{j}\omega_b)\Delta t} \\ &= E_x^2 \Delta \widehat{\chi}^{0}_{xx} + \widehat{\psi}^{1}_{xx} \mathrm{e}^{-(\nu_c - \mathrm{j}\omega_b)\Delta t},\end{aligned} \tag{A6}$$

$$\begin{aligned}\widehat{\psi}^{3}_{xx} &= E_x^3 \Delta \widehat{\chi}^{0}_{xx} + E_x^2 \Delta \widehat{\chi}^{0}_{xx} \mathrm{e}^{-(\nu_c - \mathrm{j}\omega_b)\Delta t} \\ &\quad + E_x^1 \Delta \widehat{\chi}^{0}_{xx} (\mathrm{e}^{-(\nu_c - \mathrm{j}\omega_b)\Delta t})^2 \\ &= E_x^3 \Delta \widehat{\chi}^{0}_{xx} + \widehat{\psi}^{2}_{xx} \mathrm{e}^{-(\nu_c - \mathrm{j}\omega_b)\Delta t}.\end{aligned} \tag{A7}$$

综上可归纳得到

$$\begin{aligned}\widehat{\psi}^{n}_{xx} &= \sum_{m=0}^{n-1} E_x^{n-m} \Delta \widehat{\chi}^{m}_{xx} \\ &= E_x^n \Delta \widehat{\chi}^{0}_{xx} + \mathrm{e}^{-(\nu_c - \mathrm{j}\omega_b)\Delta t} \widehat{\psi}^{n-1}_{xx}.\end{aligned} \tag{A8}$$

$\widehat{\psi}^{n}_{yxy}$，$\widehat{\psi}^{n}_{yy}$，$\widehat{\psi}^{n}_{xyx}$，$\widehat{\psi}^{n}_{zz}$ 的推导与 $\widehat{\psi}^{n}_{xx}$ 相同，在此从略.

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Three-dimensional finite-difference time-domain implementation for anisotropic dispersive medium using recursive convolution method*

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Abstract

The permittivity in the frequency domain is transformed to the time domain, and the complex electric susceptibility dyadic matrix and the complex electric displacement vector in time domain are introduced. The three-dimensional finite-difference time-domain method based on the recursive convolution principle (RC-FDTD) for the electric anisotropic dispersive medium is discussed in detail. To exemplify the availability of the RC-FDTD algorithm, the backscattering RCS of a non-magnetized plasma sphere is computed, and the numerical results are the same as that of shift operator-FDTD method, which shows that the RC-FDTD method is correct and efficient. In addition, the co-polarized and cross-polarized backscattering waves in time domain for a magnetized plasma sphere are obtained by the RC-FDTD algorithm. The results show that when the external magnetic field is implemented, the cross-polarized component appear evidently.

Keywords: recursive convolution, magnetized plasma, electromagnetic scattering, finite-difference time-domain method

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