第 57 卷 第 5 期 2008 年 5 月
1000-3290/2008/57(05)/2936-05

物 理 学 报
ACTA PHYSICA SINICA

Vol.57,No.5,May,2008

基于直接离散方式的磁化铁氧体材料电磁散射的时域有限差分方法分析*

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(2007 年 9 月 11 日收到;2007 年 9 月 19 日收到修改稿)

根据磁化铁氧体材料的磁场强度和磁感应强度之间色散的本构关系式,利用时间微分算子 $\partial/\partial t$ 和 $\mathrm{j}\omega$ 的时域和频域对应关系,将磁化铁氧体材料频域的本构关系转化为时域的本构关系,然后将时间微分算子 $\partial/\partial t$ 在时域采用直接离散的方式,得到磁场强度和磁感应强度的时域有限差分迭代式.数值结果表明,该方法易于实现,简单可行,并节约内存.

关键词:电磁散射,磁化铁氧体,时域有限差分方法,直接离散方法
PACC:4110H,5170

1.引 言

铁氧体材料,在外加磁场作用下,呈现磁各向异性.铁氧体在微波波段具有旋磁特性,是环行器和移相器等线性非互易元件中的基本材料.它们现已成为微波通讯、雷达、电子对抗等微波设备中的重要器件.另外,磁化铁氧体材料对电磁波有衰减和极化特性,所以,它也是常用的吸波材料(RAM),在国防事业中受到较大的关注.由于磁化铁氧体材料的磁导率呈各向异性,需用张量来表示,对含有磁化铁氧体材料进行严格三维分析是非常困难的.随着计算机速度的不断提高,数值算法如时域有限差分(FDTD)法成为人们求解复杂电磁场问题的重要方法.

近年来,对磁化铁氧体材料物理特性的研究成为热点之一,如文献[1,2]分别研究了磁化铁氧体材料的自旋电流及自旋极化电流的影响和作用.对磁化铁氧体材料这种复杂的色散介质的电磁散射FDTD 方法研究,其主要难点在于处理色散的本构关系,即要处理频域的本构关系过渡到时域的本构关系当中的卷积问题.文献[3]中采用了递推卷积(recursive convolution,RC)方法来处理这一难题.另外,还有的作者为了回避求卷积问题,分别采用了辅助差分方程法[4](auxiliary differential equation,ADE),Z 变换法[5](Z-transform)、移位算子法[6]等方法进行处理这个难题.笔者最近提出了一种分析磁各向异性色散介质电磁散射的 Padé-FDTD 方法[7],本文提出了另外一种新的分析磁化铁氧体材料电磁散射FDTD 方法.该方法是根据磁化铁氧体材料的磁场强度和磁感应强度之间色散的本构关系式,利用时间微分算子 $\partial/\partial t$ 和 $\mathrm{j}\omega$ 的时域和频域对应关系,将磁化铁氧体材料频域的本构关系转化为时域的本构关系,然后将时间微分算子 $\partial/\partial t$ 在时域采用直接离散的方式,得到关于磁场强度和磁感应强度的时域有限差分(DD-FDTD)迭代式,进而得到 $H\rightarrow E$,$E\rightarrow B$ 和 $B\rightarrow H$ 的 FDTD 推进式.与其他处理磁各向异性色散介质电磁散射的 FDTD 方法相比,该方法简单并且易于实现,并能节约内存.

作为验证,计算了磁化铁氧体球的后向雷达散射截面(RCS),所得结果与文献[8]结果一致.理论推导及算例表明该方法是正确有效的方法.

2.磁化铁氧体电磁散射的 FDTD 迭代式

无源麦克斯韦方程组为

* 国家自然科学基金(批准号:40674060)和江苏大学高级专业人才科研启动基金(批准号:07JDG063)资助的课题.
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$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{1}$$

$$\nabla \times \boldsymbol{H} = \varepsilon_0 \varepsilon_r \frac{\partial \boldsymbol{E}}{\partial t}, \tag{2}$$

$$\boldsymbol{B} = \mu_0 \boldsymbol{\mu}_r \boldsymbol{H}, \tag{3}$$

式中相对介电系数 ε_r 假设与频率无关，而 $\boldsymbol{\mu}_r = \boldsymbol{\mu}_r(\omega)$ 与频率有关且为一并矢，即介质电参数为各向同性，但介质磁性质为色散且各向异性. 所以用 FDTD 方法处理这种介质的电磁散射时，电场三个分量的 FDTD 迭代式与常规电场的时间推进计算公式相同.

磁场三个分量的 FDTD 迭代式，则要作出如下处理：首先利用方程(1)求出磁通量密度 B 与电场强度 E 之间的迭代式，以 x 分量为例，即

$$\begin{aligned} & B_x^{n+1/2}\left(i, j+\frac{1}{2}, k+\frac{1}{2}\right) \\ & = B_x^{n-1/2}\left(i, j+\frac{1}{2}, k+\frac{1}{2}\right) \\ & - \Delta t \cdot \left[\frac{E_z^n\left(i, j+1, k+\frac{1}{2}\right) - E_z^n\left(i, j, k+\frac{1}{2}\right)}{\Delta y}\right. \\ & \left. - \frac{E_y^n\left(i, j+\frac{1}{2}, k+1\right) - E_y^n\left(i, j+\frac{1}{2}, k\right)}{\Delta z}\right]. \end{aligned} \tag{4}$$

以上分别完成 $H \to E$ 以及 $E \to B$ 的时域推进计算. 下面根据磁各向异性介质的色散本构方程(3)，采用直接离散法来导出 $B \to H$ 的时域推进计算式.

3. 磁化铁氧体材料的离散时域本构关系的推导

当外置磁场平行于 z 轴时，饱和磁化铁氧体的磁导率张量为

$$\begin{aligned} \boldsymbol{\mu} &= \mu_0(\boldsymbol{I} + \boldsymbol{\chi}) \\ &= \mu_0 \begin{bmatrix} I + \chi_{11} & \chi_{12} & 0 \\ \chi_{21} & 1 + \chi_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \end{aligned} \tag{5}$$

式中 I 为单位矩阵，$\boldsymbol{\chi}$ 为磁化率矩阵

$$\boldsymbol{\chi}(\omega) = \begin{bmatrix} \chi_{11}(\omega) & \chi_{12}(\omega) & 0 \\ \chi_{21}(\omega) & \chi_{22}(\omega) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{6}$$

$$\begin{aligned} \chi_{11}(\omega) &= \chi_{22}(\omega) \\ &= \frac{(\omega_0 + \mathrm{j}\omega\alpha)\omega_m}{(\omega_0 + \mathrm{j}\omega\alpha)^2 - \omega^2}, \\ \chi_{12}(\omega) &= -\chi_{21}(\omega) \\ &= \frac{\mathrm{j}\omega\omega_m}{(\omega_0 + \mathrm{j}\omega\alpha)^2 - \omega^2}, \end{aligned} \tag{7}$$

上式中 $\omega_0 = \gamma H_0$，H_0 为外加磁场强度的幅值. γ 为旋磁比($\gamma = 1.76 \times 10^{10}$ rad/T)，$\omega_m = \gamma \cdot 4\pi M_s$，$M_s$ 为饱和磁化率，α 为阻尼因子.

根据方程(5)，有

$$\boldsymbol{B} = \boldsymbol{\mu}\boldsymbol{H} = \mu_0 \begin{bmatrix} 1 + \chi_{11} & \chi_{12} & 0 \\ \chi_{21} & 1 + \chi_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} \boldsymbol{H}, \tag{8}$$

记

$$\widetilde{\boldsymbol{B}} = \frac{\boldsymbol{B}}{\mu_0}, \tag{9}$$

则由(8)式有

$$\widetilde{B}_x = (1 + \chi_{11}) H_x + \chi_{12} H_y, \tag{10}$$

$$\begin{aligned} \widetilde{B}_y &= (1 + \chi_{22}) H_y + \chi_{21} H_x \\ &= (1 + \chi_{11}) H_y - \chi_{12} H_x, \end{aligned} \tag{11}$$

$$\widetilde{B}_z = H_z. \tag{12}$$

由(10)式，有

$$\begin{aligned} \widetilde{B}_x &= (1 + \chi_{11}) H_x + \chi_{12} H_y, \\ &= \left[1 + \frac{(\omega_0 + \mathrm{j}\omega\alpha)\omega_m}{(\omega_0 + \mathrm{j}\omega\alpha)^2 - \omega^2}\right] H_x \\ &\quad + \frac{\mathrm{j}\omega\omega_m}{(\omega_0 + \mathrm{j}\omega\alpha)^2 - \omega^2} H_y \\ &= \frac{(\omega_0 + \mathrm{j}\omega\alpha)^2 - \omega^2 + (\omega_0 + \mathrm{j}\omega\alpha)\omega_m}{(\omega_0 + \mathrm{j}\omega\alpha)^2 - \omega^2} H_x \\ &\quad + \frac{\mathrm{j}\omega\omega_m}{(\omega_0 + \mathrm{j}\omega\alpha)^2 - \omega^2} H_y. \end{aligned} \tag{13}$$

整理上式得

$$\begin{aligned} & [\omega_0^2 + (\mathrm{j}\omega) \cdot 2\omega_0\alpha + (\mathrm{j}\omega)^2(\alpha^2 + 1)] \widetilde{B}_x \\ & = [\omega_0^2 + \omega_0\omega_m + (\mathrm{j}\omega) \cdot (2\omega_0\alpha + \alpha\omega_m) \\ & \quad + (\mathrm{j}\omega)^2(\alpha^2 + 1)] H_x + \mathrm{j}\omega\omega_m H_y. \end{aligned} \tag{14}$$

用$\frac{\partial}{\partial t}$代替 $\mathrm{j}\omega$，将(14)式过渡到时域

$$\begin{aligned} & \omega_0^2 \widetilde{B}_x + 2\omega_0\alpha \frac{\partial \widetilde{B}_x}{\partial t} + (\alpha^2 + 1)\frac{\partial^2 \widetilde{B}_x}{\partial t^2} \\ & = (\omega_0^2 + \omega_0\omega_m) H_x + (2\omega_0\alpha + \alpha\omega_m)\frac{\partial H_x}{\partial t} \\ & \quad + (\alpha^2 + 1)\frac{\partial^2 H_x}{\partial t^2} + \omega_m \frac{\partial H_y}{\partial t}, \end{aligned} \tag{15}$$

在时域，将(15)式在 $n - \frac{1}{2}$时刻离散

$$\omega_0^2 \widetilde{B}_x^{n-1/2} + 2\omega_0\alpha \frac{\widetilde{B}_x^{n+1/2} - \widetilde{B}_x^{n-3/2}}{2\Delta t}$$

$$+\left(\alpha^{2}+1\right)\frac{\tilde{B}_{x}^{n+1/2}-2\tilde{B}_{x}^{n-1/2}+\tilde{B}_{x}^{n-3/2}}{(\Delta t)^{2}}$$

$$=\left(\omega_{0}^{2}+\omega_{0}\omega_{\mathrm{m}}\right)H_{x}^{n-1/2}$$

$$+\left(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}}\right)\frac{H_{x}^{n+1/2}-H_{x}^{n-3/2}}{2\Delta t}$$

$$+\left(\alpha^{2}+1\right)\frac{H_{x}^{n+1/2}-2H_{x}^{n-1/2}+H_{x}^{n-3/2}}{(\Delta t)^{2}}$$

$$+\omega_{\mathrm{m}}\frac{H_{y}^{n+1/2}-H_{y}^{n-3/2}}{2\Delta t}. \tag{16}$$

整理上式得

$$\left(\frac{\omega_{0}\alpha}{\Delta t}+\frac{(\alpha^{2}+1)}{(\Delta t)^{2}}\right)\tilde{B}_{x}^{n+1/2}$$

$$+\left[\omega_{0}^{2}-2\frac{(\alpha^{2}+1)}{(\Delta t)^{2}}\right]\tilde{B}_{x}^{n-1/2}$$

$$+\left[-\frac{\omega_{0}\alpha}{\Delta t}+\frac{(\alpha^{2}+1)}{(\Delta t)^{2}}\right]\tilde{B}_{x}^{n-3/2}$$

$$=\left[\frac{(\alpha^{2}+1)}{(\Delta t)^{2}}+\frac{2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}}}{2\Delta t}\right]H_{x}^{n+1/2}$$

$$+\left[\omega_{0}^{2}+\omega_{0}\omega_{\mathrm{m}}-2\frac{(\alpha^{2}+1)}{(\Delta t)^{2}}\right]H_{x}^{n-1/2}$$

$$+\left[-\frac{2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}}}{2\Delta t}+\frac{(\alpha^{2}+1)}{(\Delta t)^{2}}\right]H_{x}^{n-3/2}$$

$$+\omega_{\mathrm{m}}\frac{H_{y}^{n+1/2}-H_{y}^{n-3/2}}{2\Delta t}. \tag{17}$$

于是，$H_{x}^{n+1/2}$ 可表示为

$$H_{x}^{n+1/2}=\frac{2((\alpha^{2}+1)+\omega_{0}\alpha\Delta t)}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}\tilde{B}_{x}^{n+1/2}$$

$$+\frac{2(\omega_{0}^{2}(\Delta t)^{2}-2(\alpha^{2}+1))}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}\tilde{B}_{x}^{n-1/2}$$

$$+\frac{2((\alpha^{2}+1)-\omega_{0}\alpha\Delta t)}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}\tilde{B}_{x}^{n-3/2}$$

$$-\frac{2((\omega_{0}^{2}+\omega_{0}\omega_{\mathrm{m}})\Delta t^{2}-2(\alpha^{2}+1))}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}H_{x}^{n-1/2}$$

$$-\frac{2(\alpha^{2}+1)-(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}H_{x}^{n-3/2}$$

$$+\frac{\omega_{\mathrm{m}}\Delta t}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}H_{y}^{n-3/2}$$

$$-\frac{\omega_{\mathrm{m}}\Delta t}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}H_{y}^{n+1/2}. \tag{18}$$

同理 $H_{y}^{n+1/2}$ 可表示为

$$H_{y}^{n+1/2}=\frac{2((\alpha^{2}+1)+\omega_{0}\alpha\Delta t)}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}\tilde{B}_{y}^{n+1/2}$$

$$+\frac{2(\omega_{0}^{2}(\Delta t)^{2}-2(\alpha^{2}+1))}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}\tilde{B}_{y}^{n-1/2}$$

$$+\frac{2((\alpha^{2}+1)-\omega_{0}\alpha\Delta t)}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}\tilde{B}_{y}^{n-3/2}$$

$$-\frac{2((\omega_{0}^{2}+\omega_{0}\omega_{\mathrm{m}})\Delta t^{2}-2(\alpha^{2}+1))}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}H_{y}^{n-1/2}$$

$$-\frac{2(\alpha^{2}+1)-(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}H_{y}^{n-3/2}$$

$$-\frac{\omega_{\mathrm{m}}\Delta t}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}H_{x}^{n-3/2}$$

$$+\frac{\omega_{\mathrm{m}}\Delta t}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}H_{x}^{n+1/2}. \tag{19}$$

为了书写的方便，设

$$AA=\frac{2((\alpha^{2}+1)+\omega_{0}\alpha\Delta t)}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t};$$

$$BB=\frac{2(\omega_{0}^{2}(\Delta t)^{2}-2(\alpha^{2}+1))}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t};$$

$$CC=\frac{2((\alpha^{2}+1)-\omega_{0}\alpha\Delta t)}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t};$$

$$DD=\frac{2((\omega_{0}^{2}+\omega_{0}\omega_{\mathrm{m}})\Delta t^{2}-2(\alpha^{2}+1))}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t};$$

$$EE=\frac{2(\alpha^{2}+1)-(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t};$$

$$FF=\frac{\omega_{\mathrm{m}}\Delta t}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t};$$

$$GG=\frac{\omega_{\mathrm{m}}\Delta t}{2(\alpha^{2}+1)+(2\omega_{0}\alpha+\alpha\omega_{\mathrm{m}})\Delta t}.$$

于是(18)式和(19)式可分别表示为

$$H_{x}^{n+1/2}=AA\cdot\tilde{B}_{x}^{n+1/2}+BB\cdot\tilde{B}_{x}^{n-1/2}+CC\cdot\tilde{B}_{x}^{n-3/2}$$
$$-DD\cdot H_{x}^{n-1/2}-EE\cdot H_{x}^{n-3/2}$$
$$+FF\cdot H_{y}^{n-3/2}-GG\cdot H_{y}^{n+1/2}, \tag{20}$$

$$H_{y}^{n+1/2}=AA\cdot\tilde{B}_{y}^{n+1/2}+BB\cdot\tilde{B}_{y}^{n-1/2}+CC\cdot\tilde{B}_{y}^{n-3/2}$$
$$-DD\cdot H_{y}^{n-1/2}-EE\cdot H_{y}^{n-3/2}$$
$$-FF\cdot H_{x}^{n-3/2}+GG\cdot H_{x}^{n+1/2}. \tag{21}$$

从(20)式和(21)式可知，$H_{x}^{n+1/2}$ 和 $H_{y}^{n+1/2}$ 相互耦合，不能同时求解，所以将(21)式代入(20)式，并整理得

$$H_{x}^{n+1/2}=\frac{AA}{1+GG^{2}}\cdot\tilde{B}_{x}^{n+1/2}+\frac{BB}{1+GG^{2}}\cdot\tilde{B}_{x}^{n-1/2}$$

$$+\frac{CC}{1+GG^{2}}\cdot\tilde{B}_{x}^{n-3/2}-\frac{DD}{1+GG^{2}}\cdot H_{x}^{n-1/2}$$

$$-\left(\frac{EE-GG\cdot FF}{1+GG^{2}}\right)\cdot H_{x}^{n-3/2}+\frac{GG\cdot DD}{1+GG^{2}}\cdot H_{y}^{n-1/2}$$

$$+\left(\frac{FF+GG\cdot EE}{1+GG^{2}}\right)\cdot H_{y}^{n-3/2}-\frac{GG\cdot AA}{1+GG^{2}}\cdot B_{y}^{n+1/2}$$

$$- \frac{GG \cdot BB}{1 + GG^2} \cdot B_y^{n-1/2} - \frac{GG \cdot CC}{1 + GG^2} \cdot B_y^{n-3/2}, \tag{22}$$

$$\begin{aligned} H_y^{n+1/2} = & \frac{AA}{1 + GG^2} \cdot \widetilde{B}_y^{n+1/2} + \frac{BB}{1 + GG^2} \cdot \widetilde{B}_y^{n-1/2} \\ & + \frac{CC}{1 + GG^2} \cdot \widetilde{B}_y^{n-3/2} - \frac{DD}{1 + GG^2} \cdot H_y^{n-1/2} \\ & - \left(\frac{EE - GG \cdot FF}{1 + GG^2} \right) \cdot H_y^{n-3/2} - \frac{GG \cdot DD}{1 + GG^2} \cdot H_x^{n-1/2} \\ & - \left(\frac{FF + GG \cdot EE}{1 + GG^2} \right) \cdot H_x^{n-3/2} + \frac{GG \cdot AA}{1 + GG^2} \cdot B_x^{n+1/2} \\ & + \frac{GG \cdot BB}{1 + GG^2} \cdot B_x^{n-1/2} + \frac{GG \cdot CC}{1 + GG^2} \cdot B_x^{n-3/2}. \end{aligned} \tag{23}$$

至此，我们求得磁化铁氧体材料的磁场三分量的 FDTD 迭代式，如(22)式、(23)式和(12)式.

4. 磁化铁氧体材料的 FDTD 迭代式分析

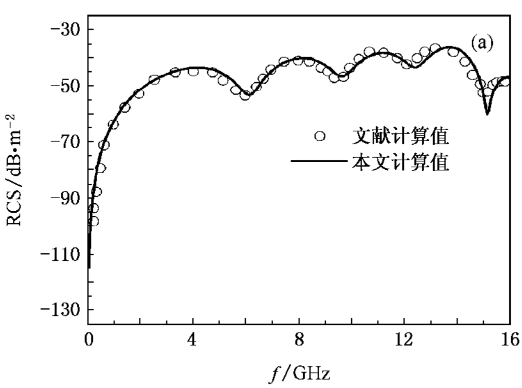
下面以磁场 x 分量 $H_x^{n+1/2}(i, j+1/2, k+1/2)$ 的计算为例，给出具体分析其迭代式. 由(22)式可知在进行空间离散时，B 与 H 的相同分量在同一节点进行离散，如 $B_x(i, j+1/2, k+1/2)$，$H_x(i, j+1/2, k+1/2)$. 但在 B 与 H 之间的离散时域的递推关系中，$H_x^{n+1/2}(i, j+1/2, k+1/2)$ 的计算不仅与当前时刻同一节点的 $B_x^{n+1/2}(i, j+1/2, k+1/2)$ 有关，而且和同一节点前两个时间步的 $B_x^{n-1/2}(i, j+1/2, k+1/2)$，$B_x^{n-3/2}(i, j+1/2, k+1/2)$，$H_x^{n-1/2}(i, j+1/2, k+1/2)$，$H_x^{n-3/2}(i, j+1/2, k+1/2)$ 相关. 另外还与前两个时刻的 $B_y^{n-1/2}$，$B_y^{n-3/2}$，$H_y^{n-1/2}$，$H_y^{n-3/2}$ 有关，但这四个分量均不在$(i, j+1/2, k+1/2)$节点，因此必须在空间上进行线性插值过渡到离散节点，例如，

$$B_y\Big|_{i,j+1/2,k+1/2}^{n-1/2} = \frac{1}{4}\Big[B_y\Big|_{i+1/2,j+1,k+1/2}^{n-1/2} + B_y\Big|_{i+1/2,j,k+1/2}^{n-1/2} + B_y\Big|_{i-1/2,j,k+1/2}^{n-1/2} + B_y\Big|_{i-1/2,j+1,k+1/2}^{n-1/2} \Big]. \tag{24}$$

磁场强度的其他两个分量的计算与此相同. 至此得到了磁化铁氧体材料本构关系在离散时域的 FDTD 表达式.

5. 数值结果

作为验证，用上述方法计算半径为 1.5 cm 的磁化铁氧体球的后向散射. FDTD 计算中设 $\delta = 0.75$ mm，$\Delta t = \delta/(2c)$，c 为光速，入射波为高斯脉冲 $E_i(t) = \exp\left[-\frac{4\pi(t - t_0)^2}{\tau^2} \right]$ 沿着 z 轴入射，其中 $\tau = 34\Delta t$ 和 $t_0 = 0.8\tau$. 外加磁场平行于 z 轴，$\omega_0 = 2\pi \times 20$ GHz，$\omega_m = 2\pi \times 10$ GHz，$\alpha = 0.1$. 计算结果如图 1 所示，其中图 1 为磁化铁氧体球后向散射的 RCS，如图 1 中的实线所示，其中图 1(a)同极化后向 RCS，(b)交叉极化后向 RCS，作为对比，图 1 中给出了文献[8]的计算值，如图中圆圈所示. 由图 1 可见，两者符合得非常好.



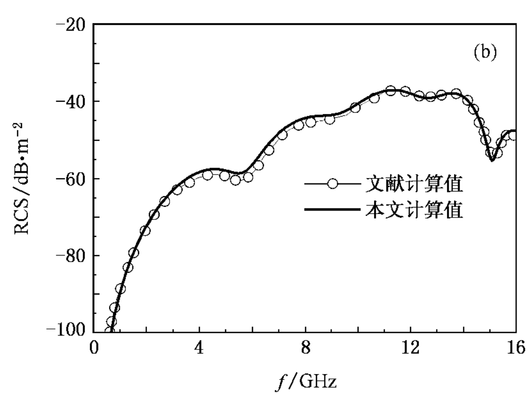


图 1　磁化铁氧体球的后向 RCS　(a) 同极化；b)交叉极化

6. 结　　论

铁氧体材料为色散介质，在外加磁场的条件下又呈现出磁各向异性. 根据磁化铁氧体的磁各向异性本构关系，采用直接法，先将 $\partial/\partial t$ 微分算子代替 $j\omega$，将本构关系过渡到时域；然后在时域进行直接离散，进而导出磁化铁氧体材料中的磁感应强度 B

和磁场强度 H 之间的色散关系在离散时域的 FDTD 表达式，从而完成了 $H\to E$，$E\to B$ 和 $B\to H$ 的时域推进计算．作为验证，计算了磁化铁氧体球的后向 RCS，所得结果与文献一致．理论推导及算例表明该方法正确可行，推导简单，概念简明，内存消耗少．

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A direct discrete-finite-difference time-domain implementation of electromagnetic scattering by magnetized ferrite medium *

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(Received 11 September 2007；revised manuscript received 19 September 2007)

Abstract

Based on the constitutive relation between the magnetic permeability and magnetic flux density，the constitutive relation of magnetized ferrite medium in frequency domain is transformed to the time domain by using the corresponding relation between the time-differential operator $\partial/\partial t$ and $\mathrm{j}\omega$. And the constitutive relation including the time-differential operator $\partial/\partial t$ in time domain is presented. By the introduction of a direct-discrete method，the time-differential operator $\partial/\partial t$ is dispersed. Then direct discrete-finite-difference time-domain (DD-FDTD) iterative formula are derived in the discrete time domain. To exemplify the availability of the algorithm，the backscattering radar scattering section of a magnetized ferrite sphere is computed，and the numerical results are the same as the reference values，and show that the DD-FDTD method is correct and efficient.

Keywords: electromagnetic scattering，magnetized ferrite，finite-difference time-domain method，direct-discrete method

PACC: 4110H，5170

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* Project supported by the National Natural Science Foundation of China (Grand No. 40674060)，and the Advanced Professional Scientific Research Foundation of Jiangsu University (Grant No. 07JDG063).

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