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Lagrange 系统 Lie 点变换下的共形不变性与守恒量*

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研究 Lagrange 系统 Lie 点变换下的共形不变性与守恒量,给出 Lagrange 系统的共形不变性定义和确定方程,讨论系统共形不变性与 Lie 对称性的关系,得到在无限小单参数点变换群作用下系统共形不变性同时是 Lie 对称性的充要条件,导出系统相应的守恒量,并给出应用算例.

关键词:Lagrange 系统,Lie 点变换,共形不变性,守恒量
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1. 引 言

动力学系统的对称性与守恒量的研究,在现代数理科学中占有重要的地位,也是分析力学的一个近代发展方向. 1918 年 Noether 研究了力学系统 Hamilton 作用量泛函在时空无限小单参数变换群作用下的不变性,发现作用量的每一种连续对称性都有一个守恒量与之对应,首次揭示了力学系统守恒量与其内在的动力学对称性之间的潜在关系,建立了 Noether 对称性理论[1,2]. 20 世纪 70 年代,人们发现并不是所有的对称性都是 Noether 的,于是出现了 Lie 对称性[3—5]. 本世纪初提出了一种新的对称性——形式不变性[6—8],人们称为 Mei 对称性[9,10]. Noether 对称性是 Hamilton 作用量在群的无限小变换下的不变性,Lie 对称性是微分方程在群的无限小变换下的一种不变性,Mei 对称性是指系统的运动方程中的动力学函数在群的无限小变换下使原方程保持形式不变. 三种对称性可直接导致守恒量,也可间接导致守恒量. 它们导致的守恒量主要有 Noether 守恒量、Hojman 守恒量和 Mei 守恒量(也称新型守恒量)[11,12]. 动力学系统的守恒量在力学研究中起着重要作用,甚至在系统的运动微分方程不可积分的情况下,某个守恒量的存在也可以使我们对所研究的局部物理状态有所了解. 因此,对称性与守恒量已成为力学领域的热门课题[13—25],文献[12]对各种约束力学系统的上述三种主要对称性与三类守恒量进行了全面、系统的研究;Galiullin 等人研究了 Birkhoff 系统的共形不变性并导出了 Noether 守恒量[26]. 本文研究 Lagrange 系统 Lie 点变换下的共形不变性及系统的守恒量,并给出一个例子说明本文结果的应用.

2. 共形不变性

对于具有 n 个自由度的 Lagrange 系统,其运动微分方程有如下形式:

$$E_s(L) = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_s} - \frac{\partial L}{\partial q_s} = 0 \quad (s = 1,\cdots,n), \tag{1}$$

式中 q_s 为系统的广义坐标,$\dot{q}_s$ 为系统的广义速度. 将(1)式展开为显形式,则有

$$F_s \equiv A_{sk}(t,\boldsymbol{q})\ddot{q}_k + B_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}) = 0 \quad (s,k = 1,\cdots,n). \tag{2}$$

若(1)式非奇异,即 $\det\left(\frac{\partial^2 L}{\partial \dot{q}_s \partial \dot{q}_k}\right) \neq 0$,则由(1)式可求得广义加速度

$$\ddot{q}_s = \alpha_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}) \qquad (s = 1,\cdots,n). \tag{3}$$

寻求 Lagrange 系统的对称性,也就是寻求方程(1)或(2)共形不变所对应的独立或非独立变量的变换集. 考虑方程(2)的对称性,为此,取时间 t 和广义

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坐标 q_s 的无限小单参数点变换群

$$t^* = t + \Delta t, q_s^*(t^*) = q_s(t) + \Delta q_s, \tag{4}$$

其展开式为

$$t^* = t + \varepsilon\xi_0(t, \boldsymbol{q}),$$
$$q_s^*(t^*) = q_s(t) + \varepsilon\xi_s(t, \boldsymbol{q}), \tag{5}$$

其中 ε 为无限小参数,ξ_0,ξ_s 为群的无限小变换的生成元或生成函数.

引入无限小生成元向量

$$X^{(0)} = \xi_0 \frac{\partial}{\partial t} + \xi_s \frac{\partial}{\partial q_s}, \tag{6}$$

其一次扩展

$$X^{(1)} = X^{(0)} + (\dot{\xi}_s - \dot{q}_s\dot{\xi}_0) \frac{\partial}{\partial \dot{q}_s}, \tag{7}$$

二次扩展

$$X^{(2)} = X^{(1)} + (\ddot{\xi}_s - 2\ddot{q}_s\dot{\xi}_0 - \dot{q}_s\ddot{\xi}_0) \frac{\partial}{\partial \ddot{q}_s}. \tag{8}$$

定义 对于非奇异矩阵 l_s^l,使得

$$X^{(2)}(F_s) = l_s^l(F_l), \tag{9}$$

则方程(2)在无限小单参数点变换(5)作用下是共形不变的,(9)式是方程(2)的共形不变的确定方程,其中 l_s^l 为共形因子.

3. 共形不变性的一般方式

为得到共形不变性的共形因子表达式,计算差值

$$X^{(2)}(F_s) - X^{(2)}(F_s)|_{F_s=0}. \tag{10}$$

由于

$$\dot{\xi}_s = \frac{\partial \xi_s}{\partial t} + \frac{\partial \xi_s}{\partial q_k}\dot{q}_k \qquad (s = 0,1,\cdots,n), \tag{11}$$

$$\ddot{\xi}_s = \frac{\partial^2 \xi_s}{\partial t^2} + 2\frac{\partial^2 \xi_s}{\partial q_k \partial t}\dot{q}_k + \frac{\partial^2 \xi_s}{\partial q_k \partial q_j}\dot{q}_k\dot{q}_j + \frac{\partial \xi_s}{\partial q_k}\ddot{q}_k$$
$$(s = 0,1,\cdots,n; k,j = 1,\cdots,n), \tag{12}$$

因此

$$\begin{aligned}
X^{(2)}(F_s) &= A_{sk}(\ddot{\xi}_k - 2\ddot{q}_k\dot{\xi}_0 - \dot{q}_k\ddot{\xi}_0) \\
&\quad + X^{(1)}(A_{sk})\ddot{q}_k + X^{(1)}(B_s) \\
&= A_{sk}(\ddot{\xi}_k - 2\ddot{q}_k\dot{\xi}_0 - \dot{q}_k\ddot{\xi}_0) \\
&\quad + X^{(0)}(A_{sk})\ddot{q}_k + X^{(0)}(B_s) \\
&\quad + (\dot{\xi}_k - \dot{q}_k\dot{\xi}_0)\frac{\partial B_s}{\partial \dot{q}_k} \\
&= A_{sk}\Big(\frac{\partial^2 \xi_k}{\partial t^2} + 2\frac{\partial^2 \xi_k}{\partial q_r \partial t}\dot{q}_r \\
&\quad + \frac{\partial^2 \xi_k}{\partial q_r \partial q_j}\dot{q}_r\dot{q}_j + \frac{\partial \xi_k}{\partial q_r}\ddot{q}_r\Big) \\
&\quad - 2A_{sk}\ddot{q}_k\Big(\frac{\partial \xi_0}{\partial t} + \frac{\partial \xi_0}{\partial q_r}\dot{q}_r\Big) \\
&\quad - A_{sk}\dot{q}_k\Big(\frac{\partial^2 \xi_0}{\partial t^2} + 2\frac{\partial^2 \xi_0}{\partial q_r \partial t}\dot{q}_r \\
&\quad + \frac{\partial^2 \xi_0}{\partial q_r \partial q_j}\dot{q}_r\dot{q}_j + \frac{\partial \xi_0}{\partial q_r}\ddot{q}_r\Big) \\
&\quad + X^{(0)}(A_{sk})\ddot{q}_k + X^{(0)}(B_s) \\
&\quad + \Big[\frac{\partial \xi_k}{\partial t} + \frac{\partial \xi_k}{\partial q_r}\dot{q}_r - \dot{q}_k\Big(\frac{\partial \xi_0}{\partial t} + \frac{\partial \xi_0}{\partial q_r}\dot{q}_r\Big)\Big]\frac{\partial B_s}{\partial \dot{q}_k} \\
&= A_{sk}\Big(\frac{\partial \xi_k}{\partial q_r} - \dot{q}_k\frac{\partial \xi_0}{\partial q_r}\Big)\ddot{q}_r \\
&\quad - 2A_{sk}\ddot{q}_k\Big(\frac{\partial \xi_0}{\partial t} + \frac{\partial \xi_0}{\partial q_r}\dot{q}_r\Big) \\
&\quad + X^{(0)}(A_{sk})\ddot{q}_k + C(t, \boldsymbol{q}, \dot{\boldsymbol{q}}),
\end{aligned} \tag{13}$$

式中 $C(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ 为其余不含 $\ddot{q}_s$ 项的代数和.同理可得

$$\begin{aligned}
&X^{(2)}(F_s)|_{F_s=0} \\
&= A_{sk}\Big(\frac{\partial \xi_k}{\partial q_r} - \dot{q}_k\frac{\partial \xi_0}{\partial q_r}\Big)\alpha_r \\
&\quad - 2A_{sk}\alpha_k\Big(\frac{\partial \xi_0}{\partial t} + \frac{\partial \xi_0}{\partial q_r}\dot{q}_r\Big) \\
&\quad + X^{(0)}(A_{sk})\alpha_k + C(t, \boldsymbol{q}, \dot{\boldsymbol{q}}),
\end{aligned} \tag{14}$$

式中 $\alpha_s = -A^{sk}B_k$.(13),(14)式有如下结果:

$$\begin{aligned}
&X^{(2)}(F_s) - X^{(2)}(F_s)|_{F_s=0} \\
&= A_{sk}\Big(\frac{\partial \xi_k}{\partial q_r} - \dot{q}_k\frac{\partial \xi_0}{\partial q_r}\Big)(\ddot{q}_r - \alpha_r) \\
&\quad - 2A_{sk}(\ddot{q}_k - \alpha_k)\Big(\frac{\partial \xi_0}{\partial t} + \frac{\partial \xi_0}{\partial q_r}\dot{q}_r\Big) \\
&\quad + X^{(0)}(A_{sk})(\ddot{q}_k - \alpha_k) \\
&\qquad (s, k, r = 1,\cdots,n).
\end{aligned} \tag{15}$$

由于

$$\begin{aligned}
\ddot{q}_k - \alpha_k &= \ddot{q}_k + A^{kl}B_l \\
&= A^{kl}(A_{lm}\ddot{q}_m + B_l) \\
&= A^{kl}F_l,
\end{aligned} \tag{16}$$

因此

$$\begin{aligned}
&X^{(2)}(F_s) - X^{(2)}(F_s)|_{F_s=0} \\
&= A_{sk}\Big(\frac{\partial \xi_k}{\partial q_r} - \dot{q}_k\frac{\partial \xi_0}{\partial q_r}\Big)A^{rl}F_l \\
&\quad - 2A_{sk}A^{kl}F_l\Big(\frac{\partial \xi_0}{\partial t} + \frac{\partial \xi_0}{\partial q_r}\dot{q}_r\Big) + X^{(0)}(A_{sk})A^{kl}F_l
\end{aligned}$$

$$= \left[A_{sk} \left(\frac{\partial \xi_k}{\partial q_r} - \dot{q}_k \frac{\partial \xi_0}{\partial q_r} \right) A^{rl} - 2\delta_s^l \left(\frac{\partial \xi_0}{\partial t} + \frac{\partial \xi_0}{\partial q_r} \dot{q}_r \right) + X^{(0)}(A_{sk}) A^{kl} \right] F_l$$

$$= \beta_s^l F_l$$

$$(s, k, r, l = 1, \cdots, n). \tag{17}$$

如果方程(2)在无限小单参数点变换(5)作用下是 Lie 对称性的，则

$$X^{(2)}(F_s)\mid_{F_s=0} = 0, \tag{18}$$

此时，由(17)和(9)式有 $l_s^l = \beta_s^l$. 若方程(2)在无限小单参数点变换(5)作用下是共形不变的，则从(17)式得到

$$(l_s^l - \beta_s^l)(F_l) = X^{(2)}(F_s)\mid_{F_s=0}$$

$$(s, l = 1, \cdots, n), \tag{19}$$

因此，方程(2)在无限小单参数点变换(5)作用下共形不变性同时是 Lie 对称性的充分与必要条件为

$$l_s^l = \beta_s^l. \tag{20}$$

由此给出以下判据.

命题 1　在无限小单参数点变换群(5)作用下，方程(2)共形不变性同时是 Lie 对称性的充分与必要条件是共形因子

$$l_s^l = A_{sk} \left(\frac{\partial \xi_k}{\partial q_r} - \dot{q}_k \frac{\partial \xi_0}{\partial q_r} \right) A^{rl} - 2\delta_s^l \left(\frac{\partial \xi_0}{\partial t} + \frac{\partial \xi_0}{\partial q_r} \dot{q}_r \right) + X^{(0)}(A_{sk}) A^{kl}$$

$$(s, k, r, l = 1, \cdots, n). \tag{21}$$

推论　若 Lagrange 方程(1)可规范为标准形式

$$F_s \equiv \ddot{q}_s - \alpha_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) = 0$$

$$(s = 1, \cdots, n), \tag{22}$$

在无限小单参数点变换群(5)作用下，方程(22)共形不变性同时是 Lie 对称性的充分与必要条件是共形因子

$$l_s^l = \frac{\partial \xi_s}{\partial q_l} - 2\dot{\xi}_0 - \dot{q}_s \frac{\partial \xi_0}{\partial q_l}. \tag{23}$$

事实上，此时(22)式相当于(2)式中 $A_{sk} = \delta_{sk}$，$B_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) = -\alpha_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$，容易由(21)式得到(23)式.

4. 共形不变性与守恒量

共形不变性满足一定条件时也可导致相应的守恒量，结果如下：

命题 2　对于满足共形因子(21)的无限小生成元 ξ_0, ξ_s，如果存在规范函数 $G = G(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ 满足如下结构方程：

$$L\dot{\xi}_0 + X^{(1)}(L) + \dot{G} = 0, \tag{24}$$

则 Lagrange 系统存在对应于共形不变性的守恒量

$$I = L\xi_0 + \frac{\partial L}{\partial \dot{q}_s}(\xi_s - \dot{q}_s \xi_0) + G = \mathrm{const}. \tag{25}$$

证明

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \dot{L}\xi_0 + L\dot{\xi}_0 + \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_s}(\xi_s - \dot{q}_s \xi_0) + \frac{\partial L}{\partial \dot{q}_s}(\dot{\xi}_s - \ddot{q}_s \xi_0 - \dot{q}_s \dot{\xi}_0) - \frac{\partial L}{\partial t}\xi_0 - \frac{\partial L}{\partial q_s}\xi_s - \frac{\partial L}{\partial \dot{q}_s}(\dot{\xi}_s - \dot{q}_s \dot{\xi}_0) - L\dot{\xi}_0$$

$$= (\xi_s - \dot{q}_s \xi_0)\left(\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_s} - \frac{\partial L}{\partial q_s} \right) = 0. \tag{26}$$

5. 算　例

二自由度系统的 Lagrange 函数

$$L = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2) - \frac{1}{2}k(q_1^2 + q_2^2)$$

$$(m, k \text{ 为常数}), \tag{27}$$

其运动微分方程为

$$\begin{aligned} F_1 &= m\ddot{q}_1 + kq_1 = 0, \\ F_2 &= m\ddot{q}_2 + kq_2 = 0. \end{aligned} \tag{28}$$

取

$$\xi_0 = 0, \xi_1 = -q_2, \xi_2 = q_1, \tag{29}$$

则

$$X^{(2)} = \xi_0 \frac{\partial}{\partial t} + \xi_s \frac{\partial}{\partial q_s} + (\dot{\xi}_s - \dot{q}_s \dot{\xi}_0)\frac{\partial}{\partial \dot{q}_s} + (\ddot{\xi}_s - 2\ddot{q}_s \dot{\xi}_0 - \dot{q}_s \ddot{\xi}_0)\frac{\partial}{\partial \ddot{q}_s}$$

$$= \xi_s \frac{\partial}{\partial q_s} + \dot{\xi}_s \frac{\partial}{\partial \dot{q}_s} + \ddot{\xi}_s \frac{\partial}{\partial \ddot{q}_s}, \tag{30}$$

$$X^{(2)}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \left(\xi_s \frac{\partial}{\partial q_s} + \dot{\xi}_s \frac{\partial}{\partial \dot{q}_s} + \ddot{\xi}_s \frac{\partial}{\partial \ddot{q}_s} \right)\begin{pmatrix} m\ddot{q}_1 + kq_1 \\ m\ddot{q}_2 + kq_2 \end{pmatrix}$$

$$= \begin{pmatrix} m\ddot{\xi}_1 + k\xi_1 \\ m\ddot{\xi}_2 + k\xi_2 \end{pmatrix} = \begin{pmatrix} -m\ddot{q}_2 - kq_2 \\ m\ddot{q}_1 + kq_1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} m\ddot{q}_1 + kq_1 \\ m\ddot{q}_2 + kq_2 \end{pmatrix}, \tag{31}$$

因此共形因子为

$$l_s^l = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{32}$$

也可从(21)式求出共形因子

$$\begin{aligned} l_s^l = {} & A_{sk}\left(\frac{\partial \xi_k}{\partial q_r} - \dot{q}_k \frac{\partial \xi_0}{\partial q_r}\right)A^{rl} \\ & - 2\delta_s^l\left(\frac{\partial \xi_0}{\partial t} + \frac{\partial \xi_0}{\partial q_r}\dot{q}_r\right) \\ & + X^{(0)}(A_{sk})A^{kl} = \frac{\partial \xi_s}{\partial q_l}. \end{aligned} \tag{33}$$

显然其结果与(32)式一致.共形不变的确定方程

$$X^{(2)}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}, \tag{34}$$

此时系统共形不变性同时是 Lie 对称性的.(29)式代入结构方程(24)得到

$$\dot{G} = 0. \tag{35}$$

可取 $G = 0$,从而守恒量(25)给出

$$I = m(q_1\dot{q}_2 - \dot{q}_1 q_2) = \text{const}. \tag{36}$$

6. 结 论

Lagrange 系统在 Lie 点变换下的共形不变性,可通过 Lie 对称性找到确定方程中的共形因子,该共形因子也就是系统的共形不变性同时是 Lie 对称性的充分与必要条件.共形不变性满足一定条件时也可导致相应的守恒量.

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Conformal invariance and conserved quantity of Lagrange systems under Lie point transformation*

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Abstract

Conformal invariance and conserved quantities of Lagrange system under Lie point transformation are studied. Firstly, the definition of conformal invariance and determining equation for the Lagrange system are provided. Secondly, the relationship between the system's conformal invariance and Lie symmetry are discussed; the necessary and sufficient condition on which the system's conformal invariance would be Lie symmetry under the infinitesimal one-parameter point transformation group is deduced; and the conserved quantities of the system are given. Lastly, an illustration example is introduced.

Keywords: Lagrange system, Lie point transformation, conformal invariance, conserved quantity

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