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非 Chetaev 型非完整系统的 Lagrange 对称性与守恒量*

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研究非 Chetaev 型非完整非保守力学系统的 Lagrange 对称性.给出了系统的 Lagrange 对称性的定义和判据,得到了非 Chetaev 型非完整非保守力学系统的 Lagrange 对称性导致守恒量(第一积分)的条件及其形式.举例说明结果的应用.

关键词:非 Chetaev 型非完整约束,Lagrange 对称性,守恒量

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1.引 言

人们在研究非线性非完整系统动力学时,常常采用虚位移的 Chetaev 定义.然而,对有些力学系统,虚位移并不满足 Chetaev 条件.1957 年,Novoselov 给出了一个非 Chetaev 型非线性非完整约束的例子,其后研究了系统的运动微分方程[1].Rumyatsev 指出,如用无惯性的随动系统来代替,则可实现非线性非完整约束.关于非 Chetaev 型非完整系统的对称性研究已取得重要进展[2—14].

1966 年,Currie 和 Saletan 研究了单自由度力学系统的等效 Lagrange 函数问题,并指出在此情形下存在守恒量[15].1981 年,Hojman 和 Harleston 将此结果推广到了一般的多自由度系统[16].赵跃宇等将这种等效 Lagrange 函数问题,即对应于某一个 Lagrange 函数的运动微分方程的每一个解都满足从另一个 Lagrange 函数得到的运动微分方程,称为 Lagrange 对称性[17],并将结果进一步推广到完整非保守系统.本文进一步研究非 Chetaev 型非完整系统的 Lagrange 对称性,得出结论:对于非 Chetaev 型非完整系统,在一定条件下,如果存在两组动力学函数 L,Q_s,Λ_s 和 $\bar{L},\bar{Q}_s,\bar{\Lambda}_s$ 导致系统相同的运动方程,则与 L 和 $\bar{L}$ 相关联的矩阵 Λ 的各次幂的迹是系统的守恒量(第一积分).文末举例说明结果的应用.

2.Lagrange 对称性的定义和判据

假设力学系统的位形由 n 个广义坐标 q_s $(s=1,\cdots,n)$ 来确定.系统的运动受有 g 个彼此独立的非 Chetaev 型非完整约束

$$f_\beta(t,\boldsymbol{q},\dot{\boldsymbol{q}})=0,(\beta=1,\cdots,g). \tag{1}$$

假设约束(1)加在虚位移 δq_s 上的条件为

$$F_{\beta s}(t,\boldsymbol{q},\dot{\boldsymbol{q}})\delta q_s=0,$$
$$(\beta=1,\cdots,g;s=1,\cdots,n), \tag{2}$$

一般说来,$F_{\beta s}$ 与 $\partial f_\beta/\partial\dot{q}_s$ 无关.特别地,当取

$$F_{\beta s}=\frac{\partial f_\beta}{\partial\dot{q}_s} \tag{3}$$

时,则退化为 Chetaev 型非完整约束.系统的运动微分方程可表为[13]

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{q}_s}-\frac{\partial L}{\partial q_s}=Q_s+\lambda_\beta F_{\beta s},$$
$$(s=1,\cdots,n), \tag{4}$$

其中 L 为系统的 Lagrange 函数,Q_s 为非势广义力,λ_β 为约束乘子.假设系统非奇异,即设

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$$\det(A_{sk}) = \det\left(\frac{\partial^2 L}{\partial \dot{q}_s \partial \dot{q}_k}\right) \neq 0, \tag{5}$$

则在运动微分方程积分以前,可由约束方程(1)和运动方程(4)求出乘子 λ_β 作为 $t,\boldsymbol{q},\dot{\boldsymbol{q}}$ 的函数,于是方程(4)可表为

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_s} - \frac{\partial L}{\partial q_s} = Q_s + \Lambda_s, (s = 1,\cdots,n), \tag{6}$$

其中 $\Lambda_s = \Lambda_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}) = \lambda_\beta F_{\beta s}$ 称为系统的广义非完整约束力.展开方程(6),记作

$$\ddot{q}_s = \alpha_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}), (s = 1,\cdots,n). \tag{7}$$

定义

$$\Gamma_s = A_{sk}\ddot{q}_k + \frac{\partial^2 L}{\partial \dot{q}_s \partial q_k}\dot{q}_k + \frac{\partial^2 L}{\partial \dot{q}_s \partial t} - \frac{\partial L}{\partial q_s} - Q_s - \Lambda_s, \tag{8}$$

如果将动力学函数 $L = L(t,\boldsymbol{q},\dot{\boldsymbol{q}})$, $Q_s = Q_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})$, $\Lambda_s = \Lambda_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ 变换为 $\bar{L} = \bar{L}(t,\boldsymbol{q},\dot{\boldsymbol{q}})$, $\bar{Q}_s = \bar{Q}_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})$, $\bar{\Lambda}_s = \bar{\Lambda}_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})$,则有

$$\bar{\Gamma}_s = \bar{A}_{sk}\ddot{q}_k + \frac{\partial^2 \bar{L}}{\partial \dot{q}_s \partial q_k}\dot{q}_k + \frac{\partial^2 \bar{L}}{\partial \dot{q}_s \partial t} - \frac{\partial \bar{L}}{\partial q_s} - \bar{Q}_s - \bar{\Lambda}_s, \tag{9}$$

其中

$$\bar{A}_{sk} = \frac{\partial^2 \bar{L}}{\partial \dot{q}_s \partial \dot{q}_k}, \det(\bar{A}_{sk}) \neq 0. \tag{10}$$

定义 对于非 Chetaev 型非完整系统(1),(4),如果由动力学函数 L, Q_s, Λ_s 确定的运动微分方程

$$\Gamma_s = 0 \tag{11}$$

的每一个解都满足由动力学函数 $\bar{L}, \bar{Q}_s, \bar{\Lambda}_s$ 确定的运动微分方程

$$\bar{\Gamma}_s = 0, \tag{12}$$

且反之亦然,则相应不变性称为系统的 Lagrange 对称性.

由(9)式和方程(12),可得

$$\ddot{q}_s = \bar{A}^{sk}\left(\frac{\partial \bar{L}}{\partial q_k} + \bar{Q}_k + \bar{\Lambda}_k - \frac{\partial^2 \bar{L}}{\partial \dot{q}_k \partial q_j}\dot{q}_j - \frac{\partial^2 \bar{L}}{\partial \dot{q}_k \partial t}\right). \tag{13}$$

将方程(13),(8)式代入方程(11),有

$$A_{sk}\bar{A}^{kj}\left(\frac{\partial \bar{L}}{\partial q_j} + \bar{Q}_j + \bar{\Lambda}_j - \frac{\partial^2 \bar{L}}{\partial \dot{q}_j \partial q_l}\dot{q}_l - \frac{\partial^2 \bar{L}}{\partial \dot{q}_j \partial t}\right) = \frac{\partial L}{\partial q_s} + Q_s + \Lambda_s - \frac{\partial^2 L}{\partial \dot{q}_s \partial q_k}\dot{q}_k - \frac{\partial^2 L}{\partial \dot{q}_s \partial t}, \tag{14}$$

于是有

判据 对于非 Chetaev 型非完整系统(1),(4),如果两组动力学函数 L, Q_s, Λ_s 和 $\bar{L}, \bar{Q}_s, \bar{\Lambda}_s$ 满足关系(14)式,则相应不变性为系统的 Lagrange 对称性.

3. Lagrange 对称性导致的守恒量

将(14)式代入(8)式,有

$$\Gamma_s = A_{sk}\bar{A}^{kj}\left(\bar{A}_{jl}\ddot{q}_l + \frac{\partial^2 \bar{L}}{\partial \dot{q}_j \partial q_l}\dot{q}_l + \frac{\partial^2 \bar{L}}{\partial \dot{q}_j \partial t} - \frac{\partial \bar{L}}{\partial q_j} - \bar{Q}_j - \bar{\Lambda}_j\right) = A_{sk}\bar{A}^{kj}\bar{\Gamma}_j, \tag{15}$$

即

$$\bar{\Gamma}_s = \Lambda_s^k \Gamma_k, \tag{16}$$

其中

$$\Lambda_s^k = \bar{A}_{sj}A^{jk}, \tag{17}$$

且 $\det\Lambda = \det(\Lambda_s^k) \neq 0$.

根据(17)式,关系式(14)式可以写成

$$\frac{\partial \bar{L}}{\partial q_s} + \bar{Q}_s + \bar{\Lambda}_s - \frac{\partial^2 \bar{L}}{\partial \dot{q}_s \partial q_l}\dot{q}_l - \frac{\partial^2 \bar{L}}{\partial \dot{q}_s \partial t} = \Lambda_s^k\left(\frac{\partial L}{\partial q_k} + Q_k + \Lambda_k - \frac{\partial^2 L}{\partial \dot{q}_k \partial q_l}\dot{q}_l - \frac{\partial^2 L}{\partial \dot{q}_k \partial t}\right). \tag{18}$$

将(18)式两边同时对 $\dot{q}_j$ 求偏导数,有

$$\frac{\partial^2 \bar{L}}{\partial q_s \partial \dot{q}_j} + \frac{\partial \bar{Q}_s}{\partial \dot{q}_j} + \frac{\partial \bar{\Lambda}_s}{\partial \dot{q}_j} - \frac{\partial^3 \bar{L}}{\partial \dot{q}_s \partial q_l \partial \dot{q}_j}\dot{q}_l - \frac{\partial^2 \bar{L}}{\partial \dot{q}_s \partial q_j} - \frac{\partial^3 \bar{L}}{\partial \dot{q}_s \partial t \partial \dot{q}_j}$$
$$= \frac{\partial \Lambda_s^k}{\partial \dot{q}_j}\left(\frac{\partial L}{\partial q_k} + Q_k + \Lambda_k - \frac{\partial^2 L}{\partial \dot{q}_k \partial q_l}\dot{q}_l - \frac{\partial^2 L}{\partial \dot{q}_k \partial t}\right) + \Lambda_s^k\left(\frac{\partial^2 L}{\partial q_k \partial \dot{q}_j} + \frac{\partial Q_k}{\partial \dot{q}_j} + \frac{\partial \Lambda_k}{\partial \dot{q}_j} - \frac{\partial^3 L}{\partial \dot{q}_k \partial q_l \partial \dot{q}_j}\dot{q}_l - \frac{\partial^2 L}{\partial \dot{q}_k \partial q_j} - \frac{\partial^3 L}{\partial \dot{q}_k \partial t \partial \dot{q}_j}\right). \tag{19}$$

根据(17)式,有

$$\Lambda_s^k A_{kl} = \bar{A}_{sl}. \tag{20}$$

将(20)式两边同时对 $\dot{q}_j$ 求偏导数,有

$$\frac{\partial \Lambda_s^k}{\partial \dot{q}_j}A_{kl} = \frac{\partial \bar{A}_{sl}}{\partial \dot{q}_j} - \Lambda_s^k\frac{\partial A_{kl}}{\partial \dot{q}_j}, \tag{21}$$

即有

$$\frac{\partial \Lambda_s^k}{\partial \dot{q}_j}A_{kl} = \frac{\partial^3 \bar{L}}{\partial \dot{q}_s \partial \dot{q}_l \partial \dot{q}_j} - \Lambda_s^k\frac{\partial^3 L}{\partial \dot{q}_k \partial \dot{q}_l \partial \dot{q}_j}$$

$$= \frac{\partial \bar{A}_{sj}}{\partial \dot{q}_l} - \Lambda_s^k \frac{\partial A_{kj}}{\partial \dot{q}_l} = \frac{\partial \Lambda_s^k}{\partial \dot{q}_l} A_{kj}. \tag{22}$$

将(20)式两边先对 q_j 求偏导数,再同乘 $\dot{q}_j$,有

$$\frac{\partial \Lambda_s^k}{\partial q_j} \frac{\partial^2 L}{\partial \dot{q}_k \partial \dot{q}_l} \dot{q}_j + \Lambda_s^k \frac{\partial^3 L}{\partial \dot{q}_k \partial \dot{q}_l \partial q_j} \dot{q}_j$$

$$= \frac{\partial^3 \bar{L}}{\partial \dot{q}_s \partial \dot{q}_l \partial q_j} \dot{q}_j. \tag{23}$$

将(20)式两边同时对 t 求偏导数,有

$$\frac{\partial \Lambda_s^k}{\partial t} A_{kl} + \Lambda_s^k \frac{\partial^3 L}{\partial \dot{q}_k \partial \dot{q}_l \partial t} = \frac{\partial^3 \bar{L}}{\partial \dot{q}_s \partial \dot{q}_l \partial t}. \tag{24}$$

将方程(7),(8)式代入方程(11),有

$$\frac{\partial L}{\partial q_s} + Q_s + \Lambda_s - \frac{\partial^2 L}{\partial \dot{q}_s \partial q_k} \dot{q}_k - \frac{\partial^2 L}{\partial \dot{q}_s \partial t} = A_{sk} \alpha_k. \tag{25}$$

将(25)式代入(19)式,并利用(22)—(24)式,得

$$\frac{\bar{d} \Lambda_s^k}{dt} A_{kj} = \frac{\partial^2 \bar{L}}{\partial q_s \partial \dot{q}_j} - \frac{\partial^2 \bar{L}}{\partial \dot{q}_s \partial q_j} + \frac{\partial \bar{Q}_s}{\partial \dot{q}_j} + \frac{\partial \bar{\Lambda}_s}{\partial \dot{q}_j}$$

$$- \Lambda_s^k \left(\frac{\partial^2 L}{\partial q_k \partial \dot{q}_j} - \frac{\partial^2 L}{\partial \dot{q}_k \partial q_j} \right)$$

$$- \Lambda_s^k \left(\frac{\partial Q_k}{\partial \dot{q}_j} + \frac{\partial \Lambda_k}{\partial \dot{q}_j} \right), \tag{26}$$

其中

$$\frac{\bar{d}}{dt} = \frac{\partial}{\partial t} + \dot{q}_j \frac{\partial}{\partial q_j} + \alpha_j \frac{\partial}{\partial \dot{q}_j}. \tag{27}$$

引进两个矩阵 $T = (T_{sj})$,$\bar{T} = (\bar{T}_{sj})$为

$$T_{sj} = \frac{\partial^2 L}{\partial q_s \partial \dot{q}_j} - \frac{\partial^2 L}{\partial \dot{q}_s \partial q_j}, \tag{28}$$

$$\bar{T}_{sj} = \frac{\partial^2 \bar{L}}{\partial q_s \partial \dot{q}_j} - \frac{\partial^2 \bar{L}}{\partial \dot{q}_s \partial q_j}. \tag{29}$$

当系统的非势广义力和广义非完整约束力满足

$$\Lambda_s^k \frac{\partial Q_k}{\partial \dot{q}_j} = \frac{\partial \bar{Q}_s}{\partial \dot{q}_j}, \quad (s, j = 1, \cdots, n), \tag{30}$$

$$\Lambda_s^k \frac{\partial \Lambda_k}{\partial \dot{q}_j} = \frac{\partial \bar{\Lambda}_s}{\partial \dot{q}_j}, \quad (s, j = 1, \cdots, n), \tag{31}$$

或者满足

$$\Lambda_s^k \frac{\partial}{\partial \dot{q}_j} (Q_k + \Lambda_k)$$

$$= \frac{\partial}{\partial \dot{q}_j} (\bar{Q}_s + \bar{\Lambda}_s), (s, j = 1, \cdots, n) \tag{32}$$

时,由(26)式得到一矩阵微分方程

$$\frac{\bar{d}}{dt} \Lambda = \bar{T} A^{-1} - \Lambda T A^{-1}, \tag{33}$$

其中 $\Lambda = \bar{A} A^{-1}$.根据(28),(29)式及 A,$\bar{A}$ 的定义可知 T,$\bar{T}$ 为反对称矩阵,A,$\bar{A}$ 为对称矩阵,利用矩阵迹的性质,得

$$\frac{\bar{d}}{dt} (\mathrm{tr} \Lambda) = \mathrm{tr} \left(\frac{\bar{d} \Lambda}{dt} \right) = 0. \tag{34}$$

由方程(33)和(34)式,容易得出

$$\frac{\bar{d}}{dt} (\mathrm{tr} \Lambda^m) = 0, \tag{35}$$

其中 m 为正整数.由(35)式,我们有

$$\mathrm{tr} \Lambda^m = \mathrm{const}. \tag{36}$$

于是有

定理　对于非 Chetaev 型非完整系统(1),(4),如果非势广义力和广义非完整约束力满足条件(30)和(31)(或满足条件(32)),则系统的 Lagrange 对称性导致形如(36)式的守恒量(第一积分).

上述定理具有普遍意义:Chetaev 型非完整系统、非保守系统[17]、Lagrange 系统[15,16] 的结果均为此定理的特例,且此定理可以进一步推广到各类约束力学系统.

4. 算　例

例 1　某力学系统的 Lagrange 函数为

$$L = \frac{1}{2} (\dot{q}_1^2 + \dot{q}_2^2) - q_1, \tag{37}$$

非势广义力为

$$Q_1 = -t, Q_2 = -\frac{\dot{q}_1}{t}, \tag{38}$$

非完整约束为

$$f = \dot{q}_2 - t\dot{q}_1 = 0, \tag{39}$$

虚位移满足条件[13]

$$\delta q_1 - \delta q_2 = 0, \tag{40}$$

试研究此非 Chetaev 型非完整系统的 Lagrange 对称性与守恒量(第一积分).

方程(4)给出

$$\ddot{q}_1 + 1 = -t + \lambda, \ddot{q}_2 = -\frac{\dot{q}_1}{t} - \lambda, \tag{41}$$

由方程(39)和(41)求得

$$\lambda = -\frac{\dot{q}_1}{t} + t, \tag{42}$$

于是有

$$\Lambda_1 = -\frac{\dot{q}_1}{t} + t, \Lambda_2 = \frac{\dot{q}_1}{t} - t. \tag{43}$$

如取

$$\bar{L} = \frac{1}{6} t \dot{q}_1^3 + \frac{1}{4} t^2 \dot{q}_1^2 + \frac{1}{2} \dot{q}_2^2,$$

$$\bar{Q}_1 = -\frac{1}{2}t\dot{q}_1, \bar{Q}_2 = 0,$$

$$\bar{\Lambda}_1 = -\frac{1}{2}\dot{q}_1^2 - \frac{1}{2}t^2, \bar{\Lambda}_2 = -t, \tag{44}$$

则有

$$(\bar{A}_{sj}) = \begin{pmatrix} t\dot{q}_1 + \frac{1}{2}t^2 & 0 \\ 0 & 1 \end{pmatrix}. \tag{45}$$

由(37)式,有

$$(A^{jk}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{46}$$

容易验证,由(37),(38),(43)和(44)式确定的两组动力学函数满足(14)式,根据本文判据,相应不变性为系统的 Lagrange 对称性.(17)式给出

$$(\Lambda_s^k) = \begin{pmatrix} t\dot{q}_1 + \frac{1}{2}t^2 & 0 \\ 0 & 1 \end{pmatrix}, \tag{47}$$

由(47)式,易验证函数 Q_s,Λ_s 和 $\bar{Q}_s$,$\bar{\Lambda}_s$ 满足条件(32),于是,由本文定理,系统存在如下第一积分:

$$I = t\dot{q}_1 + \frac{1}{2}t^2 = \text{const}. \tag{48}$$

同理,如取

$$\bar{L} = \frac{1}{2}\dot{q}_1^2 + \frac{1}{6}\dot{q}_2^3 + \frac{1}{4}t^2\dot{q}_2^2 - q_1,$$

$$\bar{Q}_1 = 0, \bar{Q}_2 = -\frac{1}{2}t^3,$$

$$\bar{\Lambda}_1 = -\frac{\dot{q}_1}{t}, \bar{\Lambda}_2 = 0, \tag{49}$$

则相应不变性为系统的 Lagrange 对称性,对应的第一积分为

$$I = \dot{q}_2 + \frac{1}{2}t^2 = \text{const}. \tag{50}$$

例 2 Novoselov 摩擦减速器[14].

问题的 Lagrange 函数为

$$L = \frac{1}{2}(I_1\dot{q}_1^2 + I_2\dot{q}_2^2), \tag{51}$$

非完整约束方程为

$$f = \left(\frac{k_1^2}{k_2^2 - \dot{q}_2^2} - K\right)\dot{q}_1 - R\dot{q}_2 = 0, \tag{52}$$

虚位移满足方程

$$\left(\frac{k_1^2}{k_2^2 - \dot{q}_2^2} - K\right)\delta q_1 - R\delta q_2 = 0, \tag{53}$$

其中 I_1, I_2, k_1, k_2, K, R 为常数.

经计算,本问题的广义非完整约束力为[14]

$$\Lambda_1 = 0, \Lambda_2 = 0. \tag{54}$$

现取

$$\bar{L} = \frac{1}{4}I_1 I_2 \dot{q}_1^2\dot{q}_2^2, \bar{\Lambda}_1 = 0, \bar{\Lambda}_2 = 0, \tag{55}$$

则有

$$(\bar{A}_{sj}) = \begin{pmatrix} \frac{1}{2}I_1 I_2\dot{q}_2^2 & I_1 I_2\dot{q}_1\dot{q}_2 \\ I_1 I_2\dot{q}_1\dot{q}_2 & \frac{1}{2}I_1 I_2\dot{q}_1^2 \end{pmatrix}. \tag{56}$$

由(51)式,有

$$(A^{jk}) = \begin{pmatrix} \frac{1}{I_1} & 0 \\ 0 & \frac{1}{I_2} \end{pmatrix}, \tag{57}$$

显然,函数 L,Λ_1,Λ_2 和 $\bar{L}$,$\bar{\Lambda}_1$,$\bar{\Lambda}_2$ 满足(14)式,由本文判据,相应不变性为此非 Chetaev 型非完整约束系统的 Lagrange 对称性.(17)式给出

$$(\Lambda_s^k) = \begin{pmatrix} \frac{1}{2}I_2\dot{q}_2^2 & I_1\dot{q}_1\dot{q}_2 \\ I_2\dot{q}_1\dot{q}_2 & \frac{1}{2}I_1\dot{q}_1^2 \end{pmatrix}, \tag{58}$$

由本文定理,系统存在守恒量,形如

$$I = \frac{1}{2}I_1\dot{q}_1^2 + \frac{1}{2}I_2\dot{q}_2^2 = \text{const}. \tag{59}$$

同理,如取

$$\bar{L} = \frac{1}{6}I_1^2\dot{q}_1^3 + \frac{1}{2}I_2\dot{q}_2^2, \bar{\Lambda}_1 = 0, \bar{\Lambda}_2 = 0, \tag{60}$$

则有守恒量

$$I = I_1\dot{q}_1 = \text{const}. \tag{61}$$

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Lagrange symmetries and conserved quantities for nonholonomic systems of non-Chetaev's type*

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Abstract

This paper focuses on Lagrange symmetries and conserved quantities for nonconservative mechanical systems with nonholonomic constraints of non-Chetaev's type. The definition and criterion of Lagrange symmetry of the systems are given. The conditions under which a Lagrange symmetry can lead to a new conserved quantity (first integral) are obtained and the form of the new conserved quantity (first integral) is presented. Two examples are given to illustrate the application of the results.

Keywords: nonholonomic system of non-Chetaev's type, Lagrange symmetry, conserved quantity

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