扰动 Vakhnenko 方程物理模型的行波解*

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研究了一类扰动 Vakhnemko 方程.给出了改进的渐近方法.首先, 对原模型系统对应的典型方程得到对应的行波解.其次, 引入一个泛函, 建立迭代关系式,将求解非线性问题转化为求解一系列的迭代序列.然后, 逐次地求出对应的解的近似式, 最后,得到了原扰动 Vakhnemko 模型行波解的任意次精度的近似展开式,并讨论了它的精度.

关键词: 泛函, 行波解, Vakhnemko 方程

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1. 引　　言

行波是非线性理论中的一个重要的内容. 它在自然科学中具有广泛的应用, 诸如流体力学, 等离子物理, 场论, 光学等[1—8]. 行波理论的研究在非线性发展方程解的研究中起着重要的作用. 近来, 出现了许多新的方法, 如双曲正切函数法, 齐次平衡法, Jacobi 椭圆函数法和辅助方程法[9]. 这些方法被应用到激波、散射光波、量子力学、神经网络、大气物理等方面, 许多学者做了大量的工作[1,2,10—16]. 目前行波理论的近似解法有新的研究, 其关键是将非线性问题转化为线性问题来处理. 本文使用的方法就是一个这样的近似方法[17]. 近来有许多近似方法被优化, 包括平均法, 边界层法, 匹配渐近展开法和多重尺度法[18—21]. 利用渐近方法作者等也研究了一类催化反应[22,23], 生态环境[24], 激波[25], 孤子[26—29], 激光脉冲[30], 海洋科学[31—34]和大气物理[35—39]等问题. 在本文中, 我们利用了一个简单而有效的技巧和迭代方法[17]研究一个广义 Vakhnenko 方程, 并得到其行波的近似解.

2. 扰动 Vakhnenko 方程

考虑如下扰动 Vakhnenko 方程[16]:

$$u_{tx} + u_x^2 + uu_{xx} + u = f(t,x,u), \qquad (1)$$

其中 f 为扰动项, 它在其自变量对应的区域内为有界的解析函数. 上述非线性方程(1)出现在等离子物理、场论等许多物理现象的理论中. 本文是来构造其行波的近似解.

首先研究广义 Vakhnenko 方程在 $f = 0$ 的情形

$$u_{tx} + u_x^2 + uu_{xx} + u = 0. \qquad (2)$$

它是一个典型的 Vakhnenko 方程. 我们能得到如下周期解[16]:

$$\bar{u}(t,x) = \frac{6k(C_1^2 + C_2^2)}{(C_1\sin(k\varepsilon_0) - C_2\cos(k\varepsilon_0))^2}, \qquad (3)$$

其中

$$x + 4k^2 t = 4k^2\varepsilon_0 + x_0 + a_0 - 6k\left[\frac{C_1\cos(k\varepsilon_0) + C_2\sin(k_0\varepsilon_0)}{C_1\sin(k\varepsilon_0) - C_2\cos(k_0\varepsilon_0)}\right],$$

且 $k > 0, a_0, x_0$ 为常数, C_1, C_2 为任意常数.

为了得到广义非线性 Vakhnenko 方程(1)的近似解, 现引入如下泛函 $F[u]$:

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$$F[u] = u - \int_{-\infty}^{x} \frac{\partial\lambda}{\partial\xi}(u - \bar{u}) d\xi - \int_{0}^{t} \frac{\partial\lambda}{\partial\tau}(u - \bar{u}) \mathrm{d}\tau - \int_{0}^{t}\int_{-\infty}^{x} \lambda \left(\frac{\partial^2 u}{\partial\tau\partial\xi} + u + \left(\frac{\partial\bar{u}}{\partial\xi}\right)^2 + \bar{u}\frac{\partial^2\bar{u}}{\partial\xi^2} - f(\tau,x,\bar{u}) \right) \mathrm{d}\tau\mathrm{d}\xi, \tag{4}$$

其中 $\bar{u}$ 为限制变量 u, 且 λ 为 Lagrange 乘子. 计算泛函(4)的变分 δF 为

$$\delta F = \delta u - \lambda\big|_{\tau = t,\xi = x}\delta u - \int_{0}^{t}\int_{-\infty}^{x} \left(\frac{\partial^2\lambda}{\partial\tau\partial\xi} + \lambda \right)\delta u \mathrm{d}\tau\mathrm{d}\xi.$$

令 $\delta F = 0$, 我们有

$$\frac{\partial^2\lambda}{\partial\tau\partial\xi} = -\lambda, \quad (\tau < t), \tag{5}$$

$$\lambda\big|_{\xi = x,\tau = t} = 1. \tag{6}$$

由(5), (6)式得

$$\lambda = \frac{1}{2}[\exp(-(x-\xi)+(t-\tau)) + \exp((x-\xi)-(t-\tau))]. \tag{7}$$

由(4), (7)式, 我们构造如下广义变分迭代:

$$\begin{aligned} u_{n+1} = u_n &- \frac{1}{2}\int_{-\infty}^{x}[\exp(-(x-\xi)+(t-\tau)) - \exp((x-\xi)-(t-\tau))](u_n - u_n)\mathrm{d}\xi \\ &+ \frac{1}{2}\int_{0}^{t}[\exp(-(x-\xi)+(t-\tau)) - \exp((x-\xi)-(t-\tau))](u_n - u_n)\mathrm{d}\tau \\ &+ \frac{1}{2}\int_{0}^{t}\int_{-\infty}^{x}[\exp(-(x-\xi)+(t-\tau)) + \exp((x-\xi)-(t-\tau))] \\ &\times \left[\frac{\partial^2 u_n}{\partial\tau\partial\xi} + u_n + \left(\frac{\partial u_n}{\partial\xi}\right)^2 + u_n\frac{\partial^2 u_n}{\partial\xi^2} - f(\tau,\xi,u_n) \right]\mathrm{d}\tau\mathrm{d}\xi. \end{aligned}$$

即

$$\begin{aligned} u_{n+1} = u_n &+ \frac{1}{2}\int_{0}^{t}\int_{-\infty}^{x}[\exp(-(x-\xi)+(t-\tau)) + \exp((x-\xi)-(t-\tau))] \\ &\times \left[\frac{\partial^2 u_n}{\partial\tau\partial\xi} + u_n + \left(\frac{\partial u_n}{\partial\xi}\right)^2 + u_n\frac{\partial^2 u_n}{\partial\xi^2} - f(\tau,\xi,u_n) \right]\mathrm{d}\tau\mathrm{d}\xi. \end{aligned} \tag{8}$$

在本文的假设下, 如果选取初始近似 $\bar{u}(t,x)$ 为非线性方程(2)的解, 则利用不动点原理[40,41] $u(t,x) = \lim\limits_{n\to\infty} u_n(t,x)$. 不难看出, $u(t,x)$ 为原 Vakhnenko 方程(1)的解. 并且近似序列 $\{u_n(t,x)\}$ 能较快的趋于精确解 $u(x,t)$.

3. 近似解

首先选取方程(1)的初始近似 u_0 为方程(2)的周期解(3). 即

$$u_0(t,x) = \frac{6k^2(C_1^2 + C_2^2)}{(C_1\sin(k\varepsilon_0) - C_2\cos(k\varepsilon_0))^2}. \tag{9}$$

将(9)式代入(8)式, 当 $n = 0$ 时, 有

$$u_1(t,x) = \frac{6k^2(C_1^2 + C_2^2)}{(C_1\sin(k\varepsilon_0) - C_2\cos(k\varepsilon_0))^2} + F(t,x), \tag{10}$$

其中

$$\begin{aligned} F(t,x) = &-\frac{1}{2}\int_{0}^{t}\int_{-\infty}^{x}[\exp(-(x-\xi)+(t-\tau)) + \exp((x-\xi)-(t-\tau))] \\ &\times \left[f\left(\tau,\xi,\frac{6k^2(C_1^2 + C_2^2)}{(C_1\sin(k\varepsilon_0) - C_2\cos(k\varepsilon_0))^2}\right) \right] \times \mathrm{d}\tau\mathrm{d}\xi. \end{aligned}$$

将(9), (10)式代入(8)式, 当 $n = 1$ 时, 得到

$$\begin{aligned} u_2(t,x) = &\frac{6k^2(C_1^2 + C_2^2)}{(C_1\sin(k\varepsilon_0) - C_2\cos(k\varepsilon_0))^2} \\ &- \frac{1}{2}\int_{0}^{t}\int_{-\infty}^{x}[\exp(-(x-\xi)+(t-\tau)) + \exp((x-\xi)-(t-\tau))] \\ &\times \left[f\left(\tau,\xi,\frac{6k^2(C_1^2 + C_2^2)}{(C_1\sin(k\varepsilon_0) - C_2\cos(k\varepsilon_0))^2}\right) \right] \times \mathrm{d}\tau\mathrm{d}\xi \\ &+ \frac{1}{2}\int_{0}^{t}\int_{-\infty}^{x}[\exp(-(x-\xi)+(t-\tau)) + \exp((x-\xi)-(t-\tau))] \\ &\times \left[\frac{\partial^2 F(\tau,\xi)}{\partial\tau\partial\xi} + F(\tau,\xi) + \frac{\partial}{\partial\xi}\frac{12k^2(C_1^2 + C_2^2)F(\tau,\xi)}{(C_1\sin(k\varepsilon_0) - C_2\cos(k\varepsilon_0))^2} + \left[\frac{\partial F(\tau,\xi)}{\partial\xi}\right]^2 \right. \\ &+ \frac{6k^2(C_1^2 + C_2^2)}{(C_1\sin(k\varepsilon_0) - C_2\cos(k\varepsilon_0))^2}\frac{\partial^2 F(\tau,\xi)}{\partial\xi^2} \\ &+ F(\tau,\xi)\frac{\partial^2}{\partial\xi^2}\left(\frac{6k^2(C_1^2 + C_2^2)}{(C_1\sin(k\varepsilon_0) - C_2\cos(k\varepsilon_0))^2}\right. \\ &+ F(\tau,\xi)\frac{\partial^2 F(\tau,\xi)}{\partial\xi^2} \end{aligned}$$

$$-f\left(\tau,\xi,\frac{6k^2(C_1^2+C_2^2)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}+F(\tau,\xi)\right)\Bigg]\mathrm{d}\zeta\mathrm{d}\xi. \tag{11}$$

同样, 我们能得到扰动 Vakhnenko 方程(1)的行波解的任意 n 次近似式 $u_n(t,x)(n\geqslant 3)$.

4. 例

作为一个特例, 我们来讨论一个扰动 Vakhnenko 方程.

设方程(1)的扰动项为 $f(t,x,u)=\varepsilon\sin^m u(0<\varepsilon\ll 1)$, 其中 $m\geqslant 2$ 为一个正整数. 则我们研究如下扰动 Vakhnenko 方程:

$$u_{tx}+u_x^2+uu_{xx}+u-\varepsilon\sin^m u=0. \tag{12}$$

首先选取

$$u_0(t,x)=\frac{6k^2(C_1^2+C_2^2)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}, \tag{13}$$

其中

$$x+4k^2t=4k^2\varepsilon_0+x_0+a_0-6k\left[\frac{C_1\cos(k\varepsilon_0)+C_2\sin(k\varepsilon_0)}{C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0)}\right],$$

且 $k>0, a_0, x_0$ 为常数, C_1, C_2 为任意常数.

利用上面的迭代方法, 由(10), (13)式, 我们有

$$u_1(t,x)=\frac{6k^2(C_1^2+C_2^2)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}+F(t,x),$$

其中

$$F(t,x)=-\frac{\varepsilon}{2}\int_0^t\int_{-\infty}^x[\exp(-(x-\xi)+(t-\tau))+\exp((x-\xi)-(t-\tau))]\times\left(\sin\frac{6k^2(C_1^2+C_2^2)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}\right)^m\times\mathrm{d}\tau\mathrm{d}\xi. \tag{14}$$

这时由(11), (14)式, 扰动 Vakhnenko 方程(12)行波解的二次近似 $u_2(t,x)$ 为

$$\begin{aligned}u_2(t,x)={}&\frac{6k^2(C_1^2+C_2^2)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}\\&-\frac{\varepsilon}{2}\int_0^t\int_{-\infty}^x[\exp(-(x-\xi)+(t-\tau))+\exp((x-\xi)-(t-\tau))]\\&\times\left(\sin\frac{6k^2(C_1^2+C_2^2)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}\right)^m\times\mathrm{d}\tau\mathrm{d}\xi\\&+\frac{1}{2}\int_0^t\int_{-\infty}^x[\exp(-(x-\xi)+(t-\tau))+\exp((x-\xi)-(t-\tau))]\\&\times\Bigg[\frac{\partial^2F(\tau,\xi)}{\partial\tau\partial\xi}+F(\tau,\xi)+\frac{\partial}{\partial\xi}\frac{12k^2(C_1^2+C_2^2)F(\tau,\xi)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}+\left[\frac{\partial F(\tau,\xi)}{\partial\xi}\right]^2\\&+\frac{6k^2(C_1^2+C_2^2)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}\frac{\partial^2F(\tau,\xi)}{\partial\xi^2}\\&+F(\tau,\xi)\frac{\partial^2}{\partial\xi^2}\left(\frac{6k^2(C_1^2+C_2^2)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}\right)+F(\tau,\xi)\frac{\partial^2F(\tau,\xi)}{\partial\xi^2}\\&-\Bigg(\sin\Bigg(\frac{6k^2(C_1^2+C_2^2)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}\\&-\frac{\varepsilon}{2}\int_0^\tau\int_{-\infty}^\xi[\exp(-(x-\xi)+(t-\tau))+\exp((x-\xi)-(t-\tau))]\\&\times\left(\sin\frac{6k^2(C_1^2+C_2^2)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}\right)^m\times\mathrm{d}\tau_1\mathrm{d}\xi_1\Bigg)^m\Bigg]\mathrm{d}\tau\mathrm{d}\xi.\end{aligned} \tag{15}$$

同样, 我们能得到扰动 Vakhnenko 方程(12)行波解的任意 n 次近似 $u_n(t,x)(n\geqslant 3)$.

5. 精度比较

由(11)式, 扰动 Vakhnenko 方程(12)的解有近似式

$$u_{\mathrm{app}}(x,t)=u_0(t,x)+F(t,x)+G(t,x), \tag{16}$$

其中 $u_0(t,x)$ 由(13)式表示, $F(t,x)$ 为

$$F(t,x)=-\frac{\varepsilon}{2}\int_0^t\int_{-\infty}^x[\exp(-(x-\xi)+(t-\tau))+\exp((x-\xi)-(t-\tau))]\times(\sin u_0(\tau,\xi))^m\mathrm{d}\xi\mathrm{d}\tau, \tag{17}$$

而 $G(t,x)$ 为

$$G(t,x)=\frac{\varepsilon}{2}\int_0^t\int_{-\infty}^x[\exp(-(x-\xi)+(t-\tau))+\exp((x-\xi)-(t-\tau))]\times\Bigg[u_0(\tau,\xi)\frac{\partial^2F(\tau,\xi)}{\partial\xi^2}$$

$$
\begin{aligned}
&+F(\tau,\xi)\frac{\partial^2 u_0(\tau,\xi)}{\partial\xi^2}\\
&+F(\tau,\xi)\frac{\partial^2 F(\tau,\xi)}{\partial\xi^2}\\
&+2\frac{\partial}{\partial\xi}(u_0(\tau,\xi)F(\tau,\xi))\\
&+\left[\frac{\partial F(\tau,\xi)}{\partial\xi}\right]^2+(\sin u_0(\tau,\xi))^m-(\sin(u_0(\tau,\xi)\\
&-\frac{\varepsilon}{2}\int_0^\tau\int_{-\infty}^\xi[\exp(-(x-\xi)+(t-\tau))\\
&+\exp((x-\xi)-(t-\tau))]\\
&\times(\sin u_0(\tau,\xi))^m\mathrm{d}\tau_1\mathrm{d}\xi_1))^m]\mathrm{d}\tau\mathrm{d}\xi. \qquad (18)
\end{aligned}
$$

由(15)—(18)式，可得

$$
\begin{aligned}
u_{\mathrm{app}}(x,t)&=u_0(t,x)+F(t,x)\\
&+\frac{\varepsilon}{2}\int_0^t\int_{-\infty}^x[\exp(-(x-\xi)+(t-\tau))\\
&+\exp((x-\xi)-(t-\tau))]\\
&\times\Big[2\frac{\partial}{\partial\xi}(u_0(\tau,\xi)F(\tau,\xi))\\
&+u_0(\tau,\xi)\frac{\partial^2 F(\tau,\xi)}{\partial\xi^2}\\
&+F(\tau,\xi)\frac{\partial^2 u_0(\tau,\xi)}{\partial\xi^2}\Big]\mathrm{d}\xi\mathrm{d}\tau\\
&+O(\varepsilon^2),\quad 0<\varepsilon\ll 1. \qquad (19)
\end{aligned}
$$

另一方面，我们用摄动方法来求得扰动 Vakhnenko 方程(12)的渐近解 $u_{asy}(t,x)$. 设

$$
u_{\mathrm{asy}}(t,x)=\sum_{i=0}^{\infty}\tilde{u}_i(t,x)\varepsilon^i. \qquad (20)
$$

将(20)式代入方程(12)，按 ε 展开非线性项，合并 ε 同次幂项，并令各次幂的系数为零. 对 ε^0 的系数为零有

$$
\tilde{u}_{0tx}+\tilde{u}_{0x}^2+\tilde{u}_0\tilde{u}_{0xx}+\tilde{u}_0=0. \qquad (21)
$$

由(2)，(3)，(13)式，(21)式有如下周期解：

$$
\begin{aligned}
\tilde{u}_0(t,x)&=u_0(t,x)\\
&=\frac{6k(C_1^2+C_2^2)}{(C_1\sin(k\varepsilon_0)-C_2\cos(k\varepsilon_0))^2}, \qquad (22)
\end{aligned}
$$

其中

$$
\begin{aligned}
x+4k^2t&=4k^2\varepsilon_0+x_0+a_0\\
&-6k\left[\frac{C_1\cos(k\varepsilon_0)+C_2\sin(k_0\varepsilon_0)}{C_1\sin(k\varepsilon_0)-C_2\cos(k_0\varepsilon_0)}\right].
\end{aligned}
$$

且 $k>0, a_0, x_0$ 为常数，C_1, C_2 为任意常数.

将(20)式代入方程(12)，按 ε 展开非线性项，合并 ε 同次幂项，并令各次幂的系数为零. 对 ε^1 的系数为零有

$$
\begin{aligned}
&\tilde{u}_{1tx}+2\tilde{u}_{0x}\tilde{u}_{1x}+\tilde{u}_0\tilde{u}_{1xx}\\
&+\tilde{u}_{0xx}\tilde{u}_1+\tilde{u}_1+(\sin\tilde{u}_0)^m=0. \qquad (23)
\end{aligned}
$$

不难看出，由(22)，(23)式，$\tilde{u}_1$ 满足积分方程

$$
\begin{aligned}
\tilde{u}_1(t,x)&=-\frac{1}{2}\int_0^t\int_{-\infty}^x[\exp(-(x-\xi)+(t-\tau))\\
&+\exp((x-\xi)-(t-\tau))]\\
&\times[2u_{0x}\tilde{u}_{1x}+u_0\tilde{u}_{1xx}+u_{0xx}\tilde{u}_1\\
&+(\sin u_0(\tau,\xi))^m]d\xi d\tau. \qquad (24)
\end{aligned}
$$

由(17)—(19)，(22)，(24)式，我们有

$$
\begin{aligned}
&\varepsilon\tilde{u}_1(t,x)-F(t,x)-G(tx)\\
&=-\frac{\varepsilon}{2}\int_0^t\int_{-\infty}^x[\exp(-(x-\xi)+(t-\tau))\\
&+\exp((x-\xi)-(t-\tau))]\\
&\times[2u_{0\xi}(\varepsilon\tilde{u}_1-F-G)_\xi\\
&+u_0(\varepsilon\tilde{u}_1-F-G)_{\xi\xi}\\
&+u_{0\xi\xi}(\varepsilon\tilde{u}_1-F-G)]\mathrm{d}\xi\mathrm{d}\tau,\\
&+O(\varepsilon^2),\quad 0<\varepsilon\ll 1. \qquad (25)
\end{aligned}
$$

对于(25)式，我们有 $\varepsilon\tilde{u}_1-F-G=O(\varepsilon^2)$，即

$$
\varepsilon\tilde{u}_1=F+G+O(\varepsilon^2),\quad 0<\varepsilon\ll 1. \qquad (26)
$$

再考虑到方程(21)的解(22)式，便可由(20)式和(26)式得到方程(12)的渐近解

$$
\begin{aligned}
u_{\mathrm{asy}}(x,t)&=u_0(t,x)+F(t,x)\\
&+\frac{\varepsilon}{2}\int_0^t\int_{-\infty}^x[\exp(-(x-\xi)\\
&+(t-\tau))+\exp((x-\xi)-(t-\tau))]\\
&\times\Big[2\frac{\partial}{\partial\xi}(u_0(\tau,\xi)F(\tau,\xi))\\
&+u_0(\tau,\xi)\frac{\partial^2 F(\tau,\xi)}{\partial\xi^2}\\
&+F(\tau,\xi)\frac{\partial^2 u_0(\tau,\xi)}{\partial\xi^2}\Big]\mathrm{d}\xi\mathrm{d}\tau\\
&+O(\varepsilon^2),\quad 0<\varepsilon\ll 1. \qquad (27)
\end{aligned}
$$

比较(19)，(27)式，它们完全相同. 由此可以看出，利用本文的迭代方法得到非线性扰动 Vakhnenko 方程的近似解 $u_{\mathrm{app}}(tx)$ 具有良好的精度.

6. 讨 论

扰动 Vakhnenko 方程的行波解是代表一类复杂的自然现象. 因此我们需要把它归化为基本模型，并用渐近方法去求解它. 本文使用的改进方法就是一个简单而有效的方法.

用本文使用的改进方法得到的近似解依赖于

初始近似 $u_0(t,x)$ 的选取. 本文中选取的是用典型方程(3)的行波解 $u_0(t,x) = \bar{u}(t,x)$. 这是很自然的. 这样能较快地趋近精确解.

用本文使用的改进方法是一个近似的解析方法, 它不同于一般的数值模拟的方法. 用本文的方法得到的近似解的展开式还能继续进行微分、积分等的解析运算. 因此, 我们还可以进一步得到其他相应物理量行波的定性、定量的性态.

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Travelling wave solution of disturbed Vakhnenko equation for physical model*

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Abstract

A kind of disturbed Vakhnemko equation is considered. The modified asymptotic method is given. Firstly, we obtain corresponding traveling wave solution of the typical Vakhnemko equation. Secondly, introducing a functional, constructing the iteration expansion of solution, the nonlinear equation is converted into a set of iteration sequence. And then, the corresponding approximations of solution are solved successively. Finally, the approximate expansion for arbitrary order accuracy of the travelling wave solution for the original disturbed Vakhnemko model is obtained and its accuracy is discussed.

Keywords: functional, travelling solution, Vakhnemko equation

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