Research on electrochemical modeling and order reduction methods for lithium-ion power batteries*

ZHANG Yangang, DONG Zeqing, ZHENG Lifeng, WANG Kai, LIANG Junfei

College of Energy and Power Engineering, North China University, Taiyuan 030051, China

Abstract. As the core power unit of new energy vehicles, the accurate modeling of power batteries is of great significance for evaluating their operating status, diagnosing faults throughout their lifecycle, and ensuring safety control under multiple operating conditions. The electrochemical model represented by the P2D model serves as a mechanistic model that can characterize the internal electrochemical reaction process of batteries on a microscale. Its accurate description of the aging and heating behavior of power batteries is an important basis for evaluating the capacity degradation, increase in internal resistance, uneven heating, and inconsistent performance of battery modules. The paper summarizes the latest advances in electrochemical modeling of lithium-ion power batteries, analyzes the coupling methods and application status of electrochemical models with equivalent circuit models, aging models, and thermal models, and focuses on the problem of numerous parameters and difficult identification of electrochemical models. In this paper, the advantages and disadvantages of the single particle model, single particle model with electrolyte, electrochemical mean model, solid-liquid phase reconstruction model, one-dimensional electrochemical model and other methods are compared with each other and analyzed for reducing the order of power battery electrochemical models, the key difficulties in characterizing electrochemical model order reduction are pointed out, and the research trends of electrochemical model reduction order reconstruction methods are prospected, in order to provide direction for the research on electrochemical model reduction order reconstruction of power batteries.

Keywords: power battery, electrochemical model, reduced order reconstruction, model coupling

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1. Introduction

As the core power device of new energy vehicles, the accurate modeling of power battery is of great significance for the operation state estimation, life cycle fault diagnosis, and multi-condition safety management and control of power battery. At present, the commonly used lithium-ion battery models mainly include equivalent circuit model, electrochemical model and data-driven model. The equivalent circuit model, as shown in Fig. 1, uses resistors, capacitors, power supplies and other components to describe the characteristics of the battery. The main advantages of the equivalent circuit model are simple structure, few control parameters and small calculation scale. However, the dependence on experimental data and the inability to effectively characterize the internal electrochemical reaction process of the battery limit its use in the characterization of the aging behavior of power batteries; The data-driven model also ignores the electrochemical reaction process inside the battery, and constructs the model by training and learning a large number of battery operation data. It has the advantages of convenient parameter identification and strong adaptive updating ability of the model, but it lacks the mechanism explanation of the physical and chemical properties of the battery and the aging heat generation behavior, and its dependence on a specific and limited training set leads to its inability to meet the application requirements of power battery management under multiple working conditions and all climates; Compared with the above two types of battery models, electrochemical models can describe the internal electrochemical reactions, ion diffusion, electrode surface reactions and other processes in detail, and can provide more accurate predictions for battery life estimation and capacity decay. However, the current electrochemical models have many parameters and are difficult to obtain. The complex model parameter identification and huge calculation scale limit the adaptive evolution of the model in the whole life cycle.

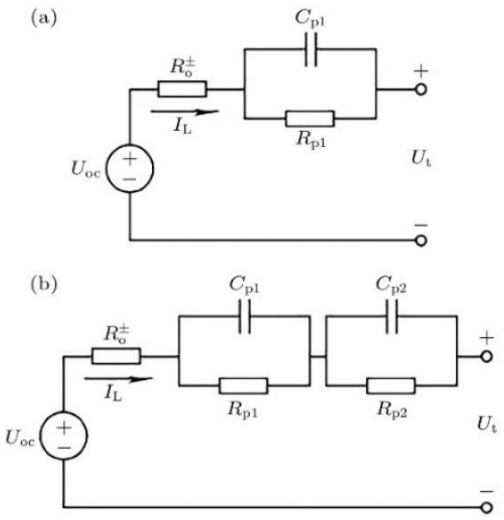


Figure 1. Equivalent-circuit model: (a) 1st order RC model; (b) 2nd order RC model.

Therefore, the electrochemical model represented by the P2D (pseudo-two-dimensions) model is used as a mechanism model to characterize the internal electrochemical reaction process of the battery from the micro-scale, and the accurate description of the aging heat generation behavior is an important basis for the evaluation of the capacity decay of the battery cell, the increase of the internal resistance, the uneven heating, and the inconsistent performance of the battery module. Scholars have done a lot of work on electrochemical modeling methods of power batteries, coupling methods of electrochemical models with models in other fields, and reduced-order reconstruction methods of electrochemical models, which have made a lot of contributions to alleviating the contradiction between the difficulty of parameter identification, computational complexity and model characterization accuracy of electrochemical models.

2. Research on Electrochemical Modeling and Application of Lithium-ion Power Battery

2.1 Lectrochemical model method of lithium ion power battery

As shown in Fig. 2, the P2D model is constructed based on the concentrated solution theory and the porous electrode theory, and is mainly composed of a porous anode and cathode, a separator, and a current collector. The model assumes that the active materials of the anode and cathode are spherical particles[1] with uniform size and regular distribution, L_p, L_n are the thicknesses of the anode and cathode respectively, L_{se} is the thickness of the separator, and the anode, cathode, and separator regions are filled with electrolyte[2]. The model employs one Butler-Volmer equation and four partial differential equations to describe the diffusion processes of lithium ions in the solid and liquid phases, as well as the potential distribution between these phases. The concentration variation of lithium ions within the active particles is defined by Fick's second law, while their transport in the liquid phase is described by the Nernst-Planck equation. The potential changes in the solid and liquid phases are defined based on charge conservation. The governing equations and boundary conditions of the model are summarized in Table 1.

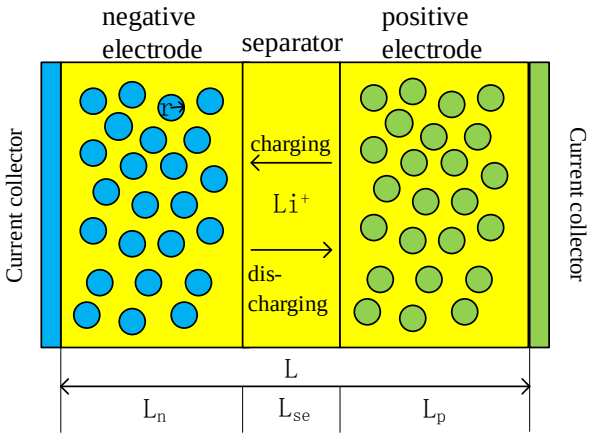


Figure 2. P2D model structure diagram.

Table 1. P2D model control equations and boundary conditions.

State	Governing equation	Boundary condition
Solid phase lithium ion concentration	$\frac{\partial c_s}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_s \frac{\partial c_s}{\partial r}\right)$	$D_s \frac{\partial c_s}{\partial r}\Big\|_{r=0} = 0, \quad D_s \frac{\partial c_s}{\partial r}\Big\|_{r=R_s} = -\frac{j}{F}$
Liquid-phase lithium ion concentration	$\varepsilon \frac{\partial c_l}{\partial t} = \frac{\partial}{\partial x}\left(D_l \frac{\partial c_l}{\partial x}\right) + (1-t^+)\frac{\nabla i_l}{F}$	$D_l \frac{\partial c_l}{\partial x}\Big\|_{x=0} = D_l \frac{\partial c_l}{\partial x}\Big\|_{x=L} = 0$
Solid phase potential	$i_s + \sigma_s \nabla \phi_s = 0$	$i_s\|_{x=0} = i_s\|_{x=L} = I$
Liquid phase potential	$i_l = -\sigma_l \nabla \phi_l + \frac{2RT\sigma_l}{F}\left(1 + \frac{\partial \ln f}{\partial \ln c_l}\right)(1-t^+)\nabla \ln c_l$	$\sigma_l \frac{\partial \phi_l}{\partial x}\Big\|_{x=0} = \sigma_l \frac{\partial \phi_l}{\partial x}\Big\|_{x=L} = 0$

The interfacial reaction rate between solid and liquid phases described by the Butler-Volmer equation is shown as (1):

$$j = i_0\left[\exp\left(\frac{\alpha_a \eta F}{RT}\right) - \exp\left(\frac{\alpha_c \eta F}{RT}\right)\right] \tag{0}$$

Where j is the reaction current density, i_0 is the exchange current density, α is the transfer constant, subscripts a and c represent the anode and cathode, respectively, and η is the overpotential. The exchange current density i_0 I

$$i_0 = Fk_0 c_l^{\alpha_a}\left(c_{s,\max} - c_{s,surf}\right)^{\alpha_a} c_{s,surf}^{\alpha_c} \tag{2}$$

Where k_0 is the reaction rate constant, $c_{s,\max}$ is the maximum lithium ion concentration in the solid phase, and $c_{s,surf}$ is the lithium ion concentration on the particle surface.

In view of the fact that the P2D electrochemical model can accurately characterize the[3] of lithium ion concentration field, potential distribution and electrochemical reaction kinetics in the battery, Feng Yi,[4], systematically studied the spatial distribution of current density and the evolution of SOC at the electrode/electrolyte interface based on the P2D model, focusing on the influence of active material components on the distribution of current density and SOC. By revealing the microscopic electrochemical reaction mechanism, the electrochemical model can further couple the multi-physical field models of battery aging, particle intercalation/deintercalation stress and microscopic heat generation to describe the aging and heat generation behavior of the battery in detail.

2.2 Coupled electrochemical-thermal modeling of lithium-ion power battery

The coupling relationship between the electrochemical model and the thermal model is shown in Fig. 3. The heat generation in the thermal model is related to the lithium ion concentration, potential, and current density in the electrochemical model, and these variables in the electrochemical model are related to the parameters closely related to temperature, such as diffusion and migration. The heat generation is used as the input of the thermal model, and the temperature T is used as the output of the thermal model to affect the parameters in the

electrochemical model, thus realizing the coupling of the electrochemical model and the thermal model.

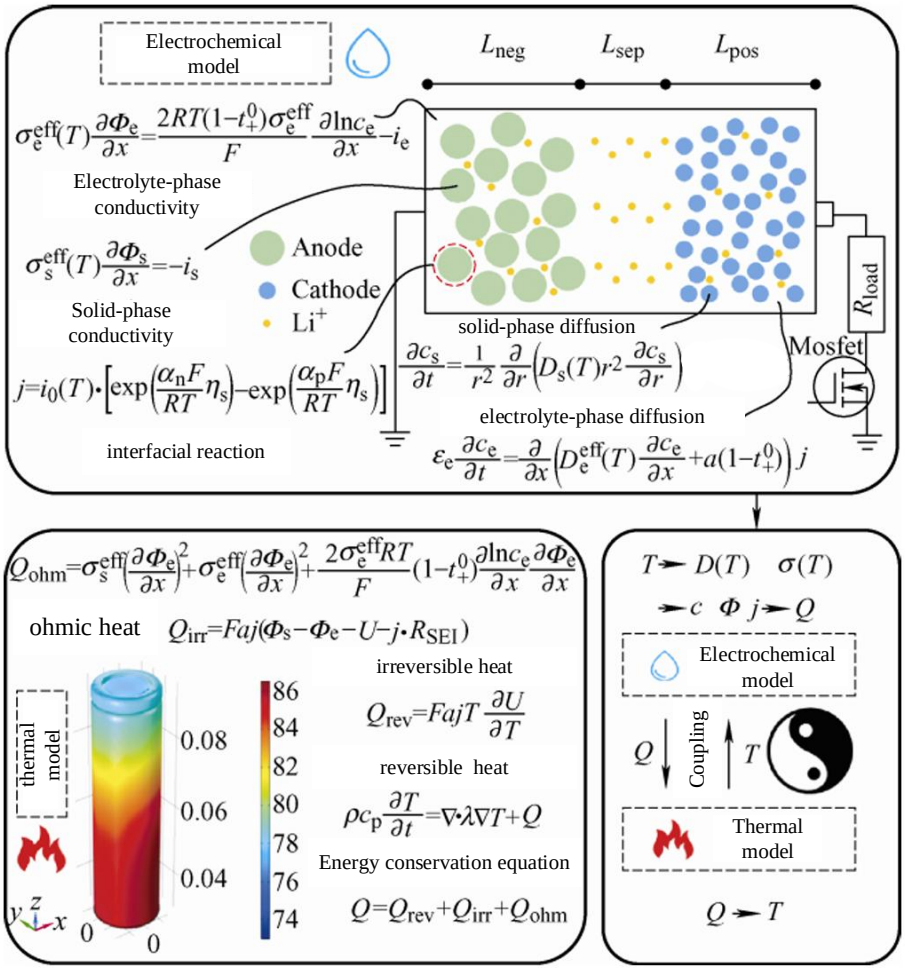


Figure 3. Electrochemical thermal model coupling relationship diagram[8].

The charge-discharge performance and service life of lithium-ion batteries are easily affected by the ambient temperature, which will cause serious safety risks such as thermal runaway, overheating and explosion[5]. Therefore, the electrochemical-thermal coupling model is often used to characterize the[6] of the heat generation and heat transfer process of power batteries, and the electrochemical-thermal coupling model is also used to effectively correct the electrochemical parameters by temperature. Jian et al. Proposed and verified the coupling method of Bernardi heat generation model and electrochemical model[7]; Nie et al.[5] verified the accuracy and applicability of the proposed electrochemical-thermal coupling model by simulating the heat generation and cell loss characteristics; The[8] of Xiong Rui et al. Used the improved P2D model to couple with the thermal model to obtain the low-temperature extreme heating model, which provided a new idea for improving the low-temperature performance of lithium-ion power batteries. The team further proposed an electrochemical-thermal coupling model[9] integrating macro, meso and micro scales, which

provided a visualization means for characterizing the process of battery capacity loss caused by lithium-ion concentration gradient; Chen Shaohui and Xiong Kai[10] extended the electrochemical-thermal coupling model from monomer scale to module scale, and described the consistency problem and heat transfer behavior of the battery pack during charging and discharging based on the electrochemical-thermal coupling model of the power battery pack; Guo et al.[11] extended the electrochemical-thermal coupling model to the coupling field with the mechanical model, accurately analyzed the microscopic heat generation and release process of the power battery, and clearly characterized the electrode structure deformation caused by the lithium ion diffusion stress. As the basic coupling model of power battery, the electrochemical-thermal coupling model is of great value to the description of the operation elements of power battery. The coupling expansion of the electrochemical-thermal coupling model with the aging, mechanical and electrical models will be the main research direction of the operation state characterization of power battery in the future.

2.3 Electrochemical-aging coupling modeling of lithium-ion power battery

Electrochemical model is often used as the basic model to describe the aging behavior because it can describe the physical and chemical reaction process inside the power battery in detail. Based on the P2D model, Atalay et al.[12] proposed an aging model to characterize the aging mechanism of SEI film and lithium coating, which can effectively predict the aging degree of power batteries under various working conditions, especially the linear and nonlinear capacity fading process; Zhang Yufei[13] constructed a characterization model of battery capacity fading by studying the growth mechanism of negative SEI film, and proposed a stepwise constant current charging strategy, which improved the charging efficiency while avoiding the growth of SEI film to a greater extent. Lithium evolution on the surface of the negative electrode is one of the key reasons for the performance degradation of the power battery under low temperature environment or large current charging conditions[14], the lithium evolution model based on the P2D model is shown in the Fig. 4, the lithium evolution side reaction is quantitatively characterized by the Bulter-Volmer equation, and the voltage generated by the side reaction is substituted into the overpotential expression of the electrochemical model to describe the surface overpotential generated by the side reaction; Based on the above lithium evolution model, Liang Fengwei et al.[15] proposed two schemes of chemical lithium evolution inhibition by adding inhibitors to the electrolyte and physical lithium evolution inhibition by controlling the charging process, which provided an effective way to solve the problem of low temperature fast charging. Thermal effect is a key factor affecting the aging behavior of power batteries. Too high or too low ambient temperature will aggravate the aging process of power batteries. The aging process and aging law of power battery were analyzed by introducing the side reaction of sei film into the electrochemical-thermal coupling model in the extended[16]; Based on the

electrochemical-thermal-mechanical coupling model, Zuo Dongxu and Li Peichao[17] pointed out that the increase of charge-discharge rate and cycle number would not only thicken the sei film, but also reduce the thermal stability of the battery. Therefore, the lithium evolution model can be used to deeply analyze the side reactions that cause battery aging, and then formulate targeted strategies to effectively deal with the impact of battery aging.

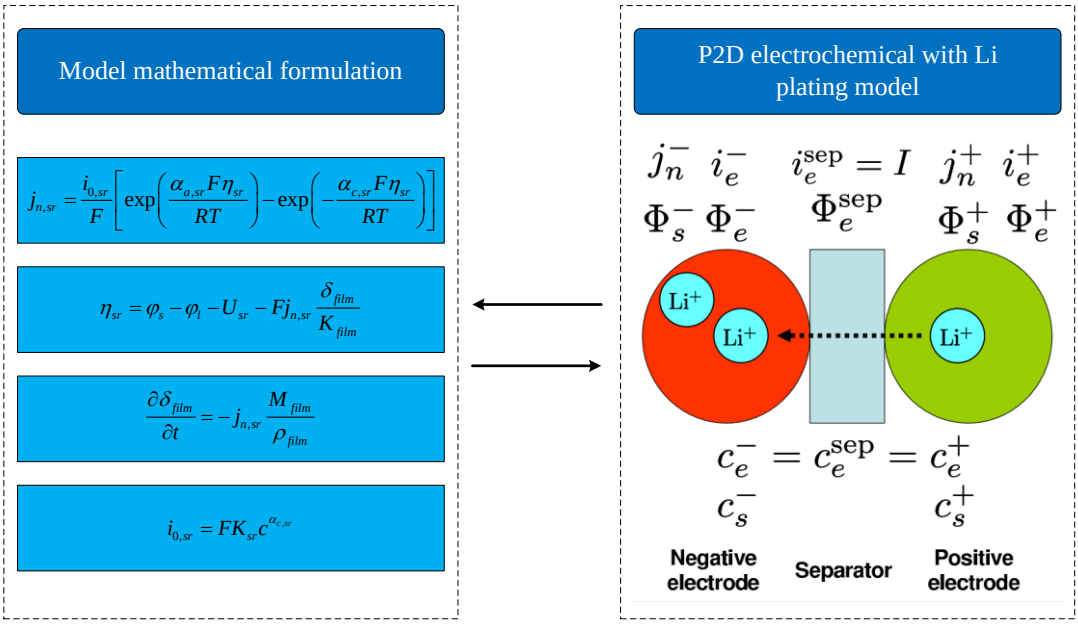


Figure 4. P2D electrochemical Li precipitation model[15].

2.4 Coupling of electrochemistry and equivalent circuit model for lithium-ion power battery

The state of charge (SOC), state of health (SOH) and state of power (SOP) of power battery are generally estimated based on ECM or P2D models, among which the electrochemical model has higher prediction accuracy[18]. Chen et al.[19] used a partial differential equation observer based on Backsteeping technique to predict battery SOC; Nicodemo et al.[20] analyzed the relationship between P2D model parameters and ECM model parameters, and used machine learning algorithm to realize the joint estimation of ECM model and P2D model parameters, which effectively improved the estimation efficiency and accuracy of power battery internal state; Based on an improved P2D model and the similarity between the transfer functions of the equivalent circuit model and the electrochemical model, Zhang et al.[21] proposed a parameter identification method for the equivalent circuit model of lithium-ion batteries considering the electrochemical characteristics. Both of these research works described the relationship between the parameters of the ECM model and those of the P2D model. Cai et al.[22] proposed a hybrid model for battery SOC prediction, which combines the advantages of the equivalent circuit model in solving speed and the electrochemical model in accuracy. In the prediction process, the Fisher information matrix is used to enhance the recursive least squares method with forgetting factor to optimize the parameter identification process. Finally, the accuracy and effectiveness of the model for SOC

prediction are verified under various dynamic conditions.

To sum up, the electrochemical model represented by the P2D model, through the coupling application with other physical fields, can not only accurately describe the operating state of the power battery, but also clearly characterize the phenomena affecting the battery capacity such as heat generation and aging of the battery cell, which plays an important role in battery state estimation, life prediction and risk assessment. However, the electrochemical model is limited by the high computational complexity in the numerical solution of the partial differential equations and the nonlinear identification of the solid-liquid interface multiphase reaction parameter system, which makes the model face the dual challenges of insufficient scalability and limited engineering applicability when facing the actual battery management system. Therefore, how to realize the reduced order reconstruction of electrochemical model on the premise of meeting the accuracy of engineering characterization is a key problem for electrochemical model to enter engineering practice.

3. Electrochemical Model Reduction and Application Status of Power Battery

The limitations of the model, such as the large number of control parameters, the difficulty of parameter identification and the large scale of calculation, are the key problems that limit the wide application of electrochemical models in engineering. In order to solve the contradiction between model complexity and computational efficiency, many scholars have used variable decoupling, equivalent reconstruction of internal physical and chemical processes, and electrochemical dimension reduction to achieve the reduction and application of electrochemical models. At present, the proposed electrochemical model reduction methods for power batteries mainly include single-particle electrode model, single-particle electrode model with liquid phase, electrochemical average value model, P2D model with solid-liquid phase reconstruction, one-dimensional electrochemical model, etc. The research and application status of these models are reviewed below.

3.1 Order reduction study based on single particle electrode model.

In the reduced-order P2D modeling framework based on physical assumptions, the Single Particle Model (SPM) stands out as one of the most representative theoretical models. Its origins can be traced back to the pioneering work of Bala et al. [23] on nickel-metal hydride battery systems. The core concept of SPM is to geometrically simplify the porous electrode structure of the positive and negative active materials into idealized spherical particles for approximate representation. Currently, this model has been applied in real-time monitoring of lithium-ion concentration within batteries [24]. Compared to the P2D model, the SPM involves fewer parameter variations and significantly lower computational complexity, demonstrating greater engineering applicability in the simulation of dynamic behavior and control strategy optimization for lithium-ion batteries. For instance, in the field of multi-physics coupling, Xu et al. [25] coupled a self-developed two-dimensional thermal model with the SPM to rapidly

predict battery thermal behavior. To meet the requirements of prediction accuracy, they divided the model into multiple temperature zones, obtained optimal parameter sets for each zone using excitation response analysis and multiple particle swarm optimization algorithms, and finally applied an adaptive extended Kalman filter algorithm for temperature estimation and parameter correction. This approach resulted in a battery thermal behavior prediction method that combines both high accuracy and computational speed.

The SPM model can realize the state estimation[26,27] of the lithium-ion battery more efficiently. Wang et al. Used a two-scale two-particle filter to adjust the parameters to obtain a more accurate SOC[28] to solve the problem of insufficient SOC estimation accuracy caused by the dynamic change of the SPM model parameters; Sun et al.[29] proposed a SOP prediction method considering the temperature effect according to the temperature dependence of SPM model parameters in a wide temperature range, which significantly enhanced the robustness of SPM model and the accuracy of SOP estimation; Cai Xue et al.[30] proposed a parameter boundary determination method based on electrochemical mechanism to solve the problem that the equivalent circuit model is easy to fall into local optimal solution and has low computational efficiency due to insufficient parameter identification accuracy in the peak power prediction of lithium batteries. The mapping relationship between the equivalent circuit elements in the equivalent circuit model and the internal mechanism of electrochemistry was clarified and verified under dynamic conditions. By comparing the robustness of each equivalent circuit model, it was found that there were nonlinear elements. At the same time, the SPM model also provides convenience for the combined application of the electrochemical model and other models. Zhao Dongmei et al.,[31], combined the advantages of the solution speed of the equivalent circuit model and the accuracy of the electrochemical model, proposed a hybrid model that takes into account the advantages of both models. From the SPM model, the topology of the equivalent circuit and the corresponding relationship between the parameters in the SPM model and the equivalent circuit elements were derived. The accuracy of the model was verified by experiments, but the error would occur in the case of high magnification In order to verify the importance of liquid phase kinetics in the electrochemical model, Pang Hui[32] constructed a reduced-order electrochemical model only considering liquid phase kinetics, and verified the accuracy of the model under multiple working conditions. Therefore, the appearance of the single particle electrode model with liquid phase expands the application range of the SPM model.

3.2 Reduced-order study based on single-particle electrode model with liquid phase

Aiming at the limitation of single particle model (SPM) in liquid phase kinetics characterization, a large number of research results show that the accuracy and prediction range of SPM model can be effectively expanded by introducing liquid phase concentration, potential and other liquid phase transport kinetics characteristic parameters to compensate

the[33,34] of SPM model. The SPM model with liquid phase kinetics equation is called single particle model with electrolyte (SPMe); Moura et al.[35] proposed a method to estimate the internal state of the battery by using the dynamic characteristics of lithium based on the SPMe model; In order to broaden the estimation range of battery internal state, Li et al.[36] proposed a method to estimate battery SOC based on SPMe model using Kalman filter algorithm, and the method can also estimate the internal parameter state of the battery. By comparison, it is found that the method can accurately estimate the internal state of the battery under the conditions of battery state uncertainty and parameter uncertainty. In order to compare the battery internal state estimation ability of SPMe model with other models, Ren Lichao[37] used SPMe model to estimate the battery SOC in the study of battery SOC estimation, which is better than the equivalent circuit model; Wang et al.[38] proposed a method for estimating the SOC of a battery based on a particle filter in the SPMe model, in which the particle swarm optimization algorithm is used to identify highly sensitive parameters. Compared with the equivalent circuit model under multiple working conditions, the SPMe model can not only characterize the electrochemical performance of the battery more efficiently, but also estimate the SOC with higher accuracy; In order to accurately estimate the internal state of the battery, including SOC, SOH and SOP, Desouza et al.[39] proposed an interconnected Sigma-point Kalman filter applied to the SPMe model, which improves the observability of some electrochemical parameters, thereby improving the estimation effect of the internal state of the battery. However, the SPMe model is obtained by reducing the order of the P2D model, so there is still a deviation between the SPMe model and the P2D model; Zhou et al.[40] further compared and analyzed the characteristics of P2D model, SPM model and SPMe model, and pointed out the advantages and disadvantages of each model. In order to further understand the cause of the error between the SPMe model and the P2D model, Xie Yizhan and Cheng Ximing[41] analyzed the error of the SPMe model from the perspective of the liquid phase, solid phase and kinetic parameters of the model, pointed out that the reaction current density was the root cause of the voltage error of the model, and proposed a series of measures to reduce the error of SPMe; Xie et al.[42] made further research on the temperature change that lithium-ion batteries will face in vehicle applications, and established a thermally coupled SPMe model that can adapt to real-time calculation and temperature change by using an electrochemical model reduction method including solid-liquid phase lithium ion mass transfer equation reduction and improved transient polarization voltage drop formula, and verified the effect of the model under dynamic conditions by comparison. Based on the excellent performance of the SPMe model, a large number of researchers have studied the coupling characteristics of the SPMe model and other physical fields. Based on the theoretical framework of electrochemical-thermal coupling of lithium-ion batteries, Xu Le et al. [43], based on the electrochemical-thermal coupling theoretical framework of lithium-ion batteries, systematically analyzed the technical bottlenecks of the traditional P2D model in practical

applications and pointed out a series of its defects. To address these shortcomings, the research team conducted a systematic comparative evaluation of three common electrochemical models from three perspectives: computational efficiency, parameter sensitivity, and prediction accuracy. As shown in Figure 5, it was found that under constant-current discharge conditions, the accuracy of the SPMe model is consistently higher than that of the SPM model across various discharge rates. Figure 6 compares the terminal voltage responses of the P2D model and the SPMe model under the Federal Urban Driving Schedule (FUDS) and the high-speed supplemental driving schedule (United States Environmental Protection Agency US06, US06). The comparison between the two models under dynamic operating conditions reveals that the SPMe model still maintains high accuracy when the current changes slowly. However, when the current fluctuates rapidly, the error of the SPMe model increases significantly. Therefore, the SPMe model remains difficult to implement effectively in engineering applications. The reason for this is that some parameters in this model are decoupled from the electrode position, preventing it from accurately describing the real internal changes of the battery under rapidly varying operating conditions.

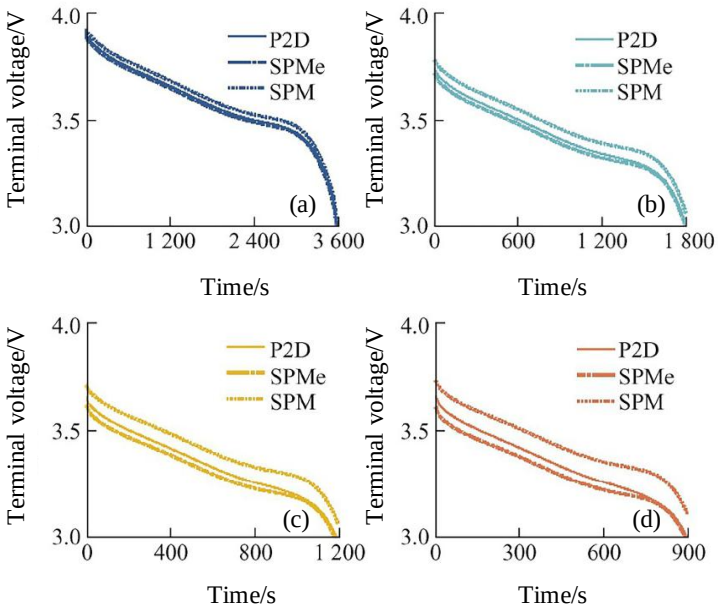


Figure 5. Comparison of constant current discharge accuracy at different ratios[43]: (a) 1 C discharge; (b) 2 C discharge; (c) 3 C discharge; (d) 4 C discharge.

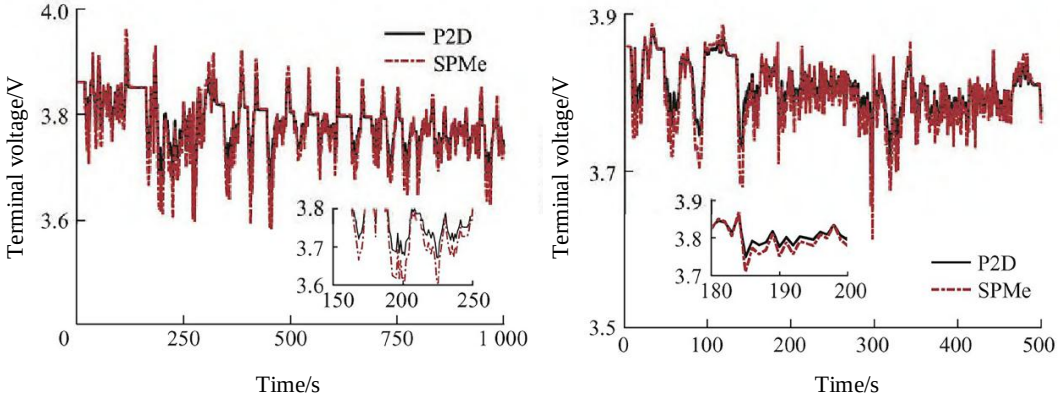


Figure 6. Comparison of model output voltage under dynamic operating conditions[43]: (a) FUDS working condition; (b) US06 working condition.

The emphasis on the liquid phase will reduce the computational efficiency of the model. In order to reduce the scale of solving the liquid lithium ion concentration in the SPMe model, Xie Yizhan and Cheng Ximing[44] assumed that the electrochemical reaction in the model only occurred at the interface between the electrolyte and the collector, and proposed a finite discrete convolution method to solve the liquid phase diffusion process under different conditions.

3.3 Reduction method based on electrochemical mean value model

Di Domenico et al.[33] proposed another kind of reduced-order electrochemical model, the electrochemical average value model, which is obtained by decoupling the parameters and X, on the basis of the battery model coupling the micro and macro; Tang et al. Developed a new method to predict the aging trajectory of battery[45], which uses machine algorithm to predict the capacity fading and internal resistance change in the aging process of battery, and proposes a feasible life extension strategy; The volume average approximation method of lithium ion diffusion in liquid phase was proposed by Kumar[46], and the equation of state of electrochemical average model was derived by Ying Zhenhua[47] based on P2D model, and the validity of the assumptions of the model was compared and analyzed. However, only the case of low discharge current density is studied. According to the assumptions, it can be inferred that when the battery works at high discharge current density and dynamic conditions, the change range of the internal parameters of the battery increases, the parameter error involved in the model will gradually increase, and the relative loss of model accuracy will also increase. Combined with the approximation condition in the electrochemical average value model, Wu et al.[48] used the Pad Padé approximation method to further simulate the solid phase diffusion process of the battery, so as to achieve the purpose of reducing the order and reconstructing the electrochemical model; Pad Padé approximation is a method to reconstruct the electrochemical model in the frequency domain. Xiong Rui and Li Xinggang[49] used this

method to simultaneously reconstruct the solid-liquid phase diffusion process, and verified the effectiveness of this method in the evaluation of lithium evolution in lithium-ion batteries. At the same time, based on the average electrochemical field theory model, the throne[50] constructs a SOC estimation system with Kalman filter architecture. Experiments show that the model has obvious errors in the large current discharge stage and vehicle BMS display. The error of the electrochemical model is essentially due to the neglect of the coupling relationship between some electrochemical parameters and electrode positions in the process of model reduction and reconstruction. In order to solve a series of problems caused by this kind of reduced-order reconstruction method, some scholars reconstruct the electrochemical model through the coupling relationship between the reduced-order parameters.

3.4 Solid-Liquid Phase Reconstruction Reduction of P2D Model and Its Application

The P2D model has the defects of complex coupling between parameters and difficult solution of the model, so some scholars reconstruct the electrochemical model through the calculation formula of the reduced-order P2D equation, such as the liquid simplified pseudo-two-dimensions (LSP2D) model. The model reconstructs the coupling relationship between the electrochemical model parameters and the electrode position X by constructing an approximate treatment mechanism of solid-liquid phase diffusion kinetics, and innovatively reconstructs the nonlinear partial differential governing equation system of the original P2D model into an ordinary differential equation system with equivalent characteristics. Subramanian et al.[51] carried out an approximate solution for the solid phase diffusion process, developed a second-order 2-parameter, a fourth-order 3-parameter, and a sixth-order 4-parameter approximate model based on second-order, fourth-order, and sixth-order polynomials, respectively, and compared the parameter changes between the approximate model and the accurate model under different reaction current densities. The results show that the accuracy of the approximate model increases with the increase of the order, but this method only solves the simple case of constant diffusion coefficient; Based on the above situation, Subramanian et al.[52] made further research based on the variable diffusion coefficient. For solid phase diffusion, they developed an approximate solution of time-dependent diffusion coefficient. In the study, three parameters, volume average concentration, surface concentration and volume concentration flux, were introduced, and dimensionless analysis of different order approximation models was carried out. Under the comparison of various test functions, it was found that high-order approximation models generally performed better; Klein et al. Used the above method to approximate the solid phase diffusion process with three parameters[53] for the reduced-order reconstruction of the electrochemical model, and Rahman et al.[54] proposed a particle swarm optimization algorithm to identify the model parameters according to the reduced-order reconstruction

method, and verified the effectiveness of the method under various charging and discharging conditions. A large number of scholars have treated the[55,56] of liquid concentration distribution and the[57] of non-uniform current distribution based on polynomial approximation, and finally obtained the reconstruction model of this kind based on polynomial approximation[58].

Li Xuhao et al.[59] used this parabolic approximation method to reconstruct the solid-liquid phase diffusion equation at the same time, coupled the reconstructed electrochemical model with the thermal model, and verified the accuracy of the model by comparing under different working conditions; Wang et al.[60] used the concentrated mass thermal model and the electrochemical model to establish the relationship between the parameters of the two models, so that the electrochemical parameters in the model could be updated according to the thermal model, and verified the accuracy of this electrochemical-thermal coupling model under different working conditions; Li et al. Used parabolic approximation to reconstruct the solid-liquid phase diffusion equation[61], and identified the model parameters based on the least squares fitting method. In the model comparison study, it was found that the overall model accuracy was higher except for the cut-off voltage stage. Fu et al.[62] also applied this method to the reduction reconstruction of the electrochemical model of sodium-ion batteries and achieved good results; Liu Zhengyu et al.[63] also reconstructed the solid-liquid phase diffusion equation with a similar parabolic approximation. In the study, the nonlinear partial differential governing equation system in the terminal voltage expression was reconstructed into an ordinary differential equation system with equivalent characteristics, and through the comparative analysis of the model curves, as shown in the Fig. 7, it was found that the polynomial approximation method had better accuracy at high rates than the single particle model, but compared with the accurate model, the liquid lithium ion concentration curve of the LSP2D model did not. All the above methods are to reduce or eliminate the coupling relationship between parameters so as to achieve the purpose of reducing the order and reconstructing the electrochemical model, and some scholars have studied the problem of excessive state quantities of electrochemical parameters caused by the dimension of the model.

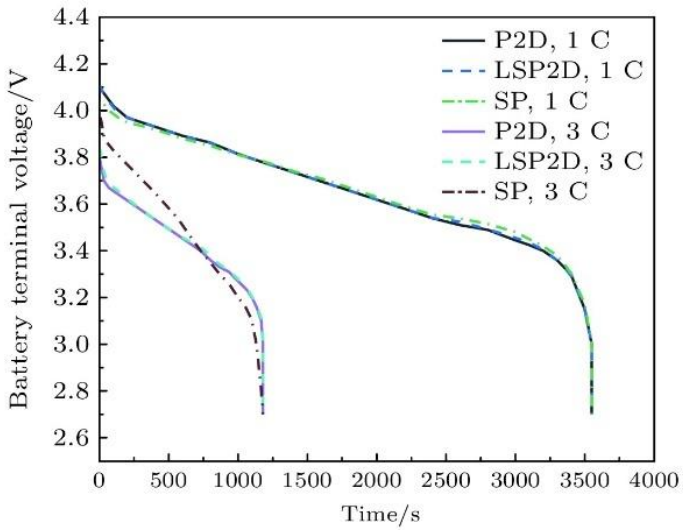


Figure 7. Discharge curves of three models at discharge rates of 1 C and 3 C[63].

3.5 Research on Order Reduction Method Based on One-dimensional Electrochemical Model

One-dimensional electrochemical model is a direct conception of electrochemical model dimension reduction, and the advantage of this model lies in the coupling with other physical field models. Kandler et al.[64] constructed a one-dimensional electrochemical model based on the dimensional reduced-order electrochemical model of the P2D model, which solved the problem of excessive state variables in the P2D model by reducing the dimension; Based on the one-dimensional electrochemical model, Lu Haoran et al. Constructed an electrochemical-thermal coupling model[65], analyzed the mechanism of different variables on the electrochemical characteristics of batteries, and quantified the heat generation contribution of each component through the heat generation decomposition method; Ma et al.[66] developed an adaptive SOC observer based on the coupling of a reduced-order one-dimensional model and a thermal model, which showed good application results in battery SOC prediction. In order to further improve the practical value of the one-dimensional electrochemical-thermal coupling model, Sangiri et al.[67] incorporated key physical laws such as mass conservation, electron conduction, charge balance and energy balance into the construction process of the one-dimensional electrochemical-thermal coupling model, so that the model can still efficiently estimate the SOC of the battery in a wide temperature range; In order to solve the core defects of P2D model in numerical calculation, such as high computational resource requirements and complex solution process, Li Guangyuan and Ma Yan[68] built an optimized one-dimensional electrochemical modeling method. In the model, Laplace transform was used to realize the attribute conversion from time domain to frequency domain, and the radial diffusion process of lithium ions in the solid phase of electrode materials was innovatively converted into the form of dimensionless admittance. In order to further improve the computational efficiency, Pad Padé approximation was used to calculate the frequency domain; Liang Xincheng et al. Deduced a one-dimensional electrochemical model from the

P2D model[69], and used the accuracy of some parameters to reflect the accuracy of the one-dimensional electrochemical model. The results show that the accuracy of the model becomes worse when the discharge current density increases. The reason is that with the increase of discharge current density, the diffusion of liquid lithium ion concentration will also accelerate, and the neglect of the coupling relationship of this parameter is the main reason for the large error of the one-dimensional model in the case of high discharge current density.

4. Research on Reduction Strategy of Power Battery Electrochemical Model

The Tab. 2 comprehensively compares the current electrochemical model reduction and reconstruction methods. The SPM model takes the solid phase lithium ion concentration, exchange current density, reaction current density, liquid phase lithium ion concentration and potential as constant values, and the model characterization accuracy is poor at high rate conditions; The SPMe model takes into account the effect of liquid phase kinetics on the basis of the SPM model, and the accuracy of the model characterization is increased at high rates, but its computational complexity is also increased compared with the SPM model. The electrochemical average value model replaces the liquid lithium ion concentration and reaction current density with a certain value, which can not accurately describe the changes of the battery under the conditions of high rate charge and discharge and variable current density, resulting in the lack of accuracy of the model. The LSP2D model approximates the governing equation describing the solid-liquid phase diffusion process with a parabolic function. Compared with the SPM model, the concentration of lithium ions in the liquid phase of the LSP2D model is still variable, so the accuracy of the LSP2d model is better than that of the SPM model under high rate conditions. However, when the working conditions change dramatically, the actual parameter variation trend seriously deviates from the parabolic trend, resulting in large model errors. In the one-dimensional electrochemical model, the liquid lithium ion concentration is regarded as a constant value. When the discharge current density increases, the actual liquid lithium ion concentration deviates from this constant value seriously, which is the main reason for the error of the model at high discharge current density.

Table 2. Comparison of simplified methods for electrochemical models.

Model name	Schematic of reduced-order electrochemical model	Reduction assumptions	Advantages	Limitations	Applicable batteries	Referen-ces
P2D model	See Appendix B figure B1	A commonly used electrochemical model; reduced-order electrochemical models are developed based on this model.	High model accuracy.	Parameter identification is challenging, the computational cost is high, and the model's applicability is limited.	This approach exhibits strong general applicability and is suitable for most lithium-ion batteries.	[10][14][16][21]
SPM model	See Appendix B figure B2	Assumption 1: The solid-phase lithium-ion concentration is uniformly distributed along the electrode thickness direction. Assumption 2: The exchange current density $i_0(x,t)$is replaced by its mean value. Assumption 3: The interfacial reaction current density $j(x,t)$at the solid–electrolyte interface is uniformly distributed along the electrode thickness direction. Assumption 4: The electrolyte-phase concentration and potential inside the cell are constant and do not vary with time.	Simple to solve; performs well at low C-rates.	Errors increase at high C-rates and under dynamic operating conditions.	Lithium-ion batteries based on low-polarization cathode materials ($LiFePO_4$) paired with graphite anodes.	[31][32]

SPMe model	See Appendix B figure B3	Assumption 1: The solid-phase lithium-ion concentration is uniformly distributed along the electrode thickness direction. Assumption 2: The exchange current density $i_0(x,t)$is replaced by its mean value. Assumption 3: The interfacial reaction current density $j(x,t)$at the solid–electrolyte interface is uniformly distributed along the electrode thickness direction.	Higher accuracy than the SPM model.	Under dynamic conditions with rapid current variations, the prediction error becomes substantial.	Liquid-electrolyte lithium-ion batteries, including ternary (NCM/NCA-type) and lithium iron phosphate (LFP) systems.	[42][44][45]
Electrochemical average-value model	See Appendix B figure B4(a)	Assumption 1: The lithium-ion concentration in the electrolyte is constant. Assumption 2: In the positive and negative electrodes, the reaction current density along the electrode thickness direction is replaced by its average value (for each electrode).	Computationally simple, with high accuracy at low C-rates.	The model cannot accurately capture battery behavior during high-rate charge/discharge or under time-varying current densities, leading to degraded accuracy.	A lithium-ion battery with a $LiCoO_2$ cathode and an MCMB2528 anode, using $LiPF_6$ as the electrolyte salt and a mixed EC/DEC solvent.	[49]
	See Appendix B figure B4(b)	Assumption 1: The reaction current density is replaced by its mean value. Assumption 2: Solid-phase diffusion in the cathode is approximated by a second-order parabola, while solid-phase diffusion in the anode is approximated by a third-order Padé approximation. Assumption 3: Electrolyte-phase diffusion is approximated using the volume-averaging method.	Accuracy increases with the order of the Padé approximation.	Increasing the approximation order requires additional state variables, thereby increasing computational complexity.	A lithium-ion cell employing a $Li_\gamma Mn_2O_4$ cathode and an Li_xC_6 anode.	[48][50]

	See Appendix B figure B5(a)	Assumption 1: The reaction current density along the electrode thickness direction is replaced by its average value. Assumption 2: The solid-phase diffusion process is described using a two-parameter parabolic approximation. Assumption 3: The electrolyte-phase diffusion process is described using a three-parameter parabolic approximation.	The reduction reconstructs the coupling among parameters, yielding higher accuracy than SPM.	When operating conditions change abruptly, the solid- and electrolyte-phase lithium-ion concentration profiles may deviate markedly from the assumed parabolic form, resulting in reduced accuracy.	Lithium iron phosphate ($LiFePO_4$, LFP) lithium-ion batteries.	[65]
LSP2D model	See Appendix B figure B5(b)	Assumption 1: The lithium-ion concentration in the electrolyte phase is constant. Assumption 2: The solid-phase diffusion process is represented by a three-parameter approximate model.	The simple computations make parameter identification easier.	The electrolyte-phase dynamics are captured with insufficient accuracy.	18650-format ternary lithium-ion batteries (e.g., NCM/NCA-based).	[55][56]
	See Appendix B figure B5(c)	Assumption 1: The effect of non-uniform reaction distribution within the electrode is neglected, and the reaction current density is approximated by the operating current density. Assumption 2: Each electrode is approximated by a single particle; the solid-phase concentration follows a three-parameter parabolic profile, and the electrolyte-phase concentration follows a two-parameter parabolic approximation. Assumption 3: The two electrodes have the same degree of reaction polarization and the same solid-phase diffusion process. Assumption 4: The influence of the internal cell temperature on model parameters is neglected.	Good overall accuracy; it can also establish the relationship between internal cell current density and external-circuit current density.	Large deviations are mainly observed near the cutoff-voltage region.	A lithium-ion battery with a $LiCoO_2$ cathode and a graphite anode.	[63][64]

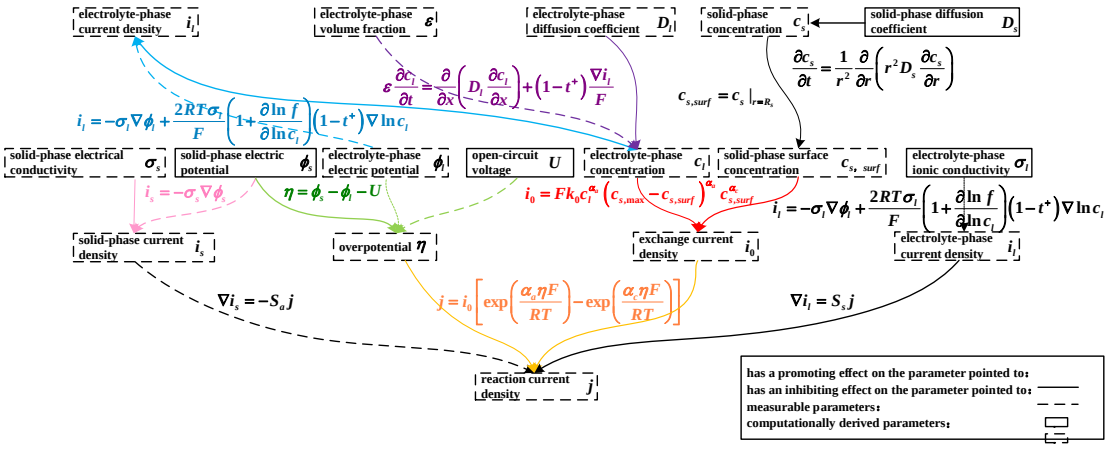
One-dimensional model	See Appendix B figure B6	Assumption 1: Parameters are constant. Assumption 2: The interfacial lithium-ion reaction current density j at the solid–electrolyte interface is decoupled from the electrolyte lithium-ion concentration c_l, and c_l is assumed to be constant.	Lower dimensionality with fewer state variables, making the model easier to solve.	For complex scenarios such as high discharge current densities or fast charge/discharge, a one-dimensional formulation cannot faithfully represent the cell's internal transient response, and the accuracy deteriorates.	A lithium-ion battery with a $LiCoO_2$ cathode and an MCMB2528 anode.	[70]

In the current study, the reduced-order electrochemical models, except the SPMe model, all reduce the order of the change of the liquid lithium ion concentration. The LSP2D model only alleviates the coupling relationship between the liquid lithium ion concentration and other parameters, while the other reduced-order electrochemical models decouple the liquid lithium-ion concentration from the parameters in the model. The accuracy of the SPMe model is greater than that of the LSP2D model and the other reduced-order models at high rate, which further verifies that the control parameters describing the liquid kinetic process are more effective for the. In addition, except for the LSP2D model, the other reduced-order models ignore the coupling relationship between the reaction current density and the parameters in the model, while the reaction current density is highly coupled with the core parameter of the battery (terminal voltage), and the accuracy of the electrochemical mechanism model is greatly affected by the change of the parameter.

To sum up, the electrochemical model order reduction methods proposed at present mainly focus on model dimension reduction and model parameter equivalent reduction, and clarifying the coupling relationship between electrochemical model characterization parameters and defining the calculation method of key electrochemical parameters are the necessary prerequisites for power battery electrochemical model order reduction analysis. Li Tao et al.[70] pointed out that the electrochemical model can be successfully applied in practice only by order reduction, aiming at the problem that the parameters of P2D model are difficult to measure or unmeasurable in practical engineering applications, and the problem that the solution of P2D model is time-consuming due to the complex coupling relationship between parameters; Kong et al. Proposed a series of reduction strategies for the complexity of electrochemical models[71], and demonstrated the feasibility of each reduction strategy in terms of efficiency and accuracy; Li Haotian et al pointed out[72] that the large number of parameters and the complexity of parameter coupling are the main reasons why the electrochemical model can not meet the real-time and efficiency requirements of online simulation. In addition, the simplification of a large number of parameters in the process of model reduction and reconstruction will inevitably introduce approximation errors. How to identify and reasonably simplify the parameters with low sensitivity to the system output response is a key problem to be solved in depth. Aiming at the problem of difficult parameter identification of P2D model, Xu et al.[73] obtained a series of highly sensitive parameters through global parameter sensitivity analysis. The research results show that the reduced order reconstruction electrochemical model based on highly sensitive parameters can reduce the calculation; Zhang Bi[74] uses parameter sensitivity analysis to reduce the workload of model parameter identification, and proposes a method to judge parameter sensitivity by observing the dispersion degree of parameter curve, which provides a new way for parameter sensitivity analysis of power battery electrochemical model.

In light of the requirement for parameter correlation in the dimensionality reduction and

equivalent simplification of electrochemical models, this paper combines the governing equations and boundary conditions of the P2D model with the parameter mapping relationships of electrochemical reaction processes to establish an electrochemical model characterization parameter correlation map, as shown in Figure 8. This map outlines the interrelationships and influence mechanisms among parameters such as exchange current density, overpotential, reaction current density, solid-phase current density, liquid-phase current density, solid-phase surface concentration, liquid-phase concentration, liquid-phase potential, open-circuit voltage, solid-phase potential, liquid-phase volume fraction, and solid-phase diffusion coefficient. Furthermore, by integrating existing literature findings, the measurement and calculation processes of key electrochemical parameters (as shown in Table 3) as well as the unique parameters and their physical meanings in various electrochemical models (as shown in Table A1 in Appendix A) are analyzed. The analysis results indicate that only a very small number of electrochemical model parameters can be directly obtained through testing. Moreover, such parameters not only require expensive testing equipment like electrochemical workstations and operation by specialized technical personnel, but a large number of electrochemical model parameters also need to be identified through complex computations. Therefore, simplifying the identification process of electrochemical parameters is also of significant value for the order reduction and reconstruction of electrochemical models. The complex nonlinear coupling relationships among the parameters of electrochemical models significantly restrict their application in real-world operating conditions. Constructing efficient reduced-order reconstructed electrochemical models is of great importance for advancing the real-time state estimation of electrochemical models in battery management systems.



Note: Except for the black lines, lines in other colors indicate that multiple parameters affect the parameter indicated by the arrow; parameters at the ends of lines of the same color all influence the parameter indicated by the arrow.

Figure 8. Coupling relationship diagram between various parameters.

Table 3. Calculation methods for some key electrochemical parameters.

Parameter category	Parameter name	Test method	Test equipment	Estimation method
Kinetic parameters	Solid-phase diffusion coefficient	Galvanostatic intermittent titration technique	charge–discharge tester	Testing
	Electrolyte-phase diffusion coefficient	Galvanostatic intermittent titration technique	charge–discharge tester	Calculated
	Electrolyte ionic conductivity	Four-electrode measurement	Conductivity meter	Measured
	Solid-phase electronic conductivity	Galvanostatic intermittent titration technique	charge–discharge tester	Calculated
	Reaction rate constant	Electrochemical impedance spectroscopy	Electrochemical workstation	Measured
	Reaction current density	Cyclic voltammetry	Electrochemical workstation	Calculated
	Exchange current density	Cyclic voltammetry	Electrochemical workstation	Calculated
	Solid-phase current density	Cyclic voltammetry	Electrochemical workstation	Calculated
	Electrolyte-phase current density	Cyclic voltammetry	Electrochemical workstation	Calculated
Concentration-field parameters	Solid-phase concentration	Galvanostatic intermittent titration technique	charge–discharge tester	Calculated
	Solid-phase surface concentration	Galvanostatic intermittent titration technique	charge–discharge tester	Calculated
	Electrolyte concentration	Cyclic voltammetry	Electrochemical workstation	Calculated
	Electrolyte volume fraction	Nuclear magnetic resonance (NMR) measurement	NMR spectrometer	Calculated
Electrical parameters	Overpotential	Difference method	Electrochemical workstation	Calculated
	Electrolyte-phase potential	Microelectrode measurement	Electrochemical workstation	Calculated
	Solid-phase potential	Three-electrode cell configuration	Electrochemical workstation	Measured
	Open-circuit voltage	Direct voltmeter measurement	Voltmeter	Measured

5. Research Trend of Electrochemical Model Reduction and Reconstruction

The electrochemical model represented by the P2D model is a mechanism model that can characterize the internal electrochemical reaction process of the battery from the microscopic scale. Its accurate description of the aging and heat generation behavior of the power battery is an important basis for the evaluation of the capacity attenuation of the battery monomer, the increase of the internal resistance, the uneven heating and the inconsistent performance of the battery module. On the one hand, the effective order reduction of the electrochemical model can significantly improve the simulation efficiency and engineering applicability of the electrochemical model, on the other hand, it can effectively reduce the coupling difficulty between the electrochemical model and other physical fields, and provide strong support for the real-time and accurate description of the operation state of the power battery and the dynamic management and control optimization. Therefore, it is an unavoidable technical requirement to carry out efficient electrochemical model reduction and reconstruction research in the field of power battery applications.

1) Focus on the electrochemical model reduction and reconstruction method for the application of multi-physical field coupling characterization, and effectively expand the application field of electrochemical mechanism model.

Although the existing electrochemical model reduction methods significantly improve the computational efficiency of multi-physical field coupling by simplifying the number of particles in the porous electrode, porosity and other microstructure parameters, the lack of characterization of the electrode microstructure deformation process leads to theoretical defects in the cross-scale coupling mechanism between the electrochemical governing equations and the temperature field and mechanical field. The weakening of the multi-scale correlation makes the model have obvious defects in predicting the thermal runaway and life decay mechanism of the battery. In order to solve the problem of multi-physical field coupling modeling, it is necessary to systematically integrate the cross-scale coupling mechanism between key physical field parameters such as thermal expansion coefficient, stress-strain equation and electrochemical constitutive equation. In the process of model reduction, it is necessary to focus on the reconstruction strategy of time-domain dimensional model based on polynomial approximation expansion, especially for the reconstruction of electrochemical sensitive parameters with significant field-field coupling characteristics, which can effectively improve the prediction accuracy of the reduced electrochemical mechanism model.

2) Strengthen the exploration of electrochemical model reduction and reconstruction methods driven by big data, and focus on solving the problem of rapid and high-precision characterization of power battery operation state under high rate conditions.

The application advantages of the proposed electrochemical model reduction method under low rate conditions have been fully proved, but the deterioration of prediction accuracy under high rate conditions has become a key bottleneck restricting the engineering application

of electrochemical models. The problems of battery monomer capacity attenuation, internal resistance increase, uneven heating and inconsistent battery module performance caused by power battery aging and heat generation behavior mostly occur under high rate discharge conditions. Therefore, it is of great significance to study the reduction and reconstruction method of electrochemical model of power battery for high rate application, which can fully describe the aging behavior of power battery under multiple operating conditions. Taking advantage of the data-driven method and the machine learning algorithm, which do not rely on model predefinition and presupposition, the data-driven model is integrated in the electrochemical model reduction and reconstruction process to give full play to the timeliness advantage of the data-driven method under high-dimensional nonlinear conditions, which provides an effective way to finally solve the problem of rapid and high-precision characterization of the operating state of power batteries under high-rate conditions.

3) The reduced-order reconstruction method of electrochemical model for real-time and fast simulation of on-board running state is studied, and the reduced-order reconstruction electrochemical model with both state representation accuracy and simulation efficiency is established.

The high-precision advantages of the electrochemical model in characterizing the micro-electrochemical mechanism reaction process of the power battery and describing the operating state of the power battery have been demonstrated. The current proposed reduced-order reconstruction methods are mostly based on laboratory data for verification, ignoring the problem of poor real-time updating ability of the electrochemical model state under on-board operating conditions. The above problems lead to significant limitations in the engineering application of the electrochemical reduced-order model under on-board complex operating conditions. Therefore, due to the limitations of the assumptions in the electrochemical model reduction process, most of the reduced order reconstruction models are currently only suitable for quasi-static analysis in the laboratory environment, and it is difficult to accurately characterize the characteristic parameters such as dynamic parameter changes, wide range temperature gradients, and complex aging mechanisms faced by the power battery in real vehicle operation. How to reveal the mapping mechanism between on-board measured parameters and electrochemical model sensitive parameters, how to reduce the coupling relationship between electrochemical parameters by using time domain reconstruction methods such as polynomial approximation, and then build an electrochemical model reduction and reconstruction framework for real-time and rapid simulation of on-board operation state, and realize high-precision battery state parameter estimation (such as state of charge, health state, power state, etc.) By embedding adaptive observation algorithm, so as to optimize the multi-objective collaborative control strategy of battery management system.

To sum up, the electrochemical model, as the basic model that can characterize the mechanism reaction of the power battery from the micro level, can accurately describe the

operation state and operation change process of the power battery by coupling other physical field models, but its huge calculation scale and complex parameter identification process seriously limit the application of the model in engineering practice. It is an unavoidable important technical requirement in the field of power battery application to carry out efficient research on electrochemical model reduction and reconstruction, focusing on the electrochemical model reduction and reconstruction method for multi-physical field coupling characterization application, the electrochemical model reductionand reconstruction method driven by big data, and the electrochemical model reductionand reconstruction method for real-time and rapid simulation of vehicle operation state, breaking through the key technology of rapid and high-precision characterization of power battery operation state under high rate conditions, and establishing a dual-purpose state.

Appendix A.

The different model parameters and their physical meaning and origin are shown in Table A1..

Table A1. Parameters of different models and their physical meanings and sources

Model	Parameter	Physical meaning	Source	Impact on model accuracy
SPM+SPMe	Average solid-phase lithium-ion concentration	The overall solid-phase lithium-ion concentration is represented by an average value, thereby neglecting concentration variations across different electrode positions.	[34]	At high C-rates, neglecting the transient evolution of the solid-phase lithium-ion concentration further induces deviations in the predicted overpotential.
	Average exchange current density	The exchange current density is described using an average value, ignoring its spatial heterogeneity along the electrode.	[35][44]	At high C-rates, the actual exchange current density can deviate substantially from its averaged value, leading to errors in the predicted polarization voltage.
SPM + SPMe + electrochemical-averaged model	Average reaction current density	The reaction current density is approximated by an average value, which neglects spatial variations across the electrode.	[44][34] [35][49]	Under high-rate operation, local variations in overpotential are neglected, which in turn degrades the accuracy of the model-predicted terminal voltage.
SPM + electrochemical-averaged model	Average electrolyte lithium-ion concentration	The electrolyte lithium-ion concentration is represented by an average value, and its transient evolution is neglected.	[44][49]	At high C-rates, electrolyte concentration gradients become pronounced; averaging tends to underestimate concentration polarization, thereby biasing the output voltage prediction.
LSP2D	Volume-averaged concentration	This term denotes the volume-averaged solid-phase lithium-ion concentration at the microscopic particle scale.	[53][54]	This directly affects the steady-state capacity prediction and may introduce bias in state-of-charge (SOC) estimation.
	Surface concentration	This term describes the lithium-ion concentration at the surface of solid-phase particles.		This influences the calculation of exchange current density and polarization voltage, and consequently affects the predicted terminal voltage.
	Volume-averaged concentration flux	This term characterizes the averaged spatial variation of the solid-phase lithium-ion concentration within the system.		Closely coupled to concentration gradients, this can lead to inaccuracies in predicting diffusion-induced polarization.

Appendix B. Reduced order electrochemical model diagram

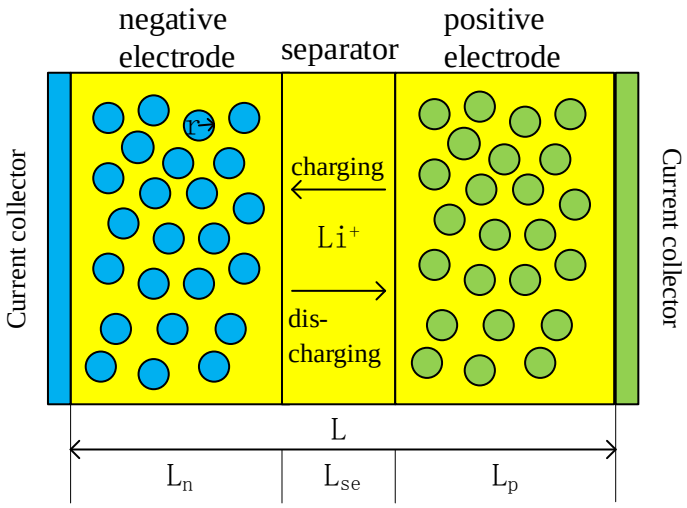


Figure B1. P2D model

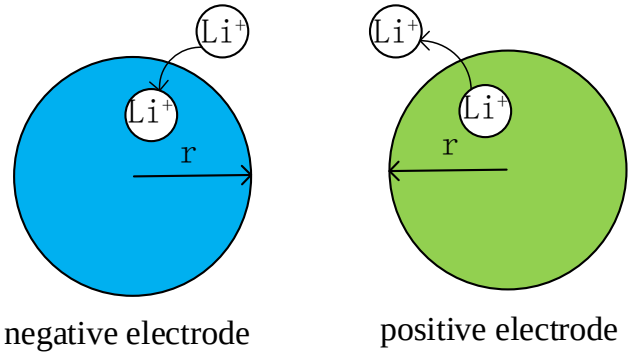


Figure B2. SPM model

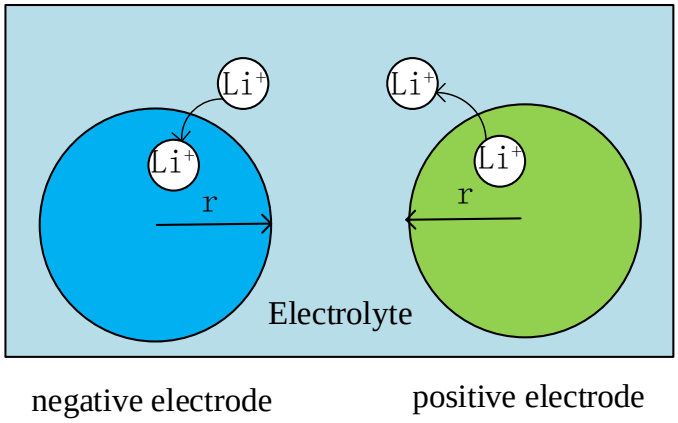


Figure B3. SPMe model

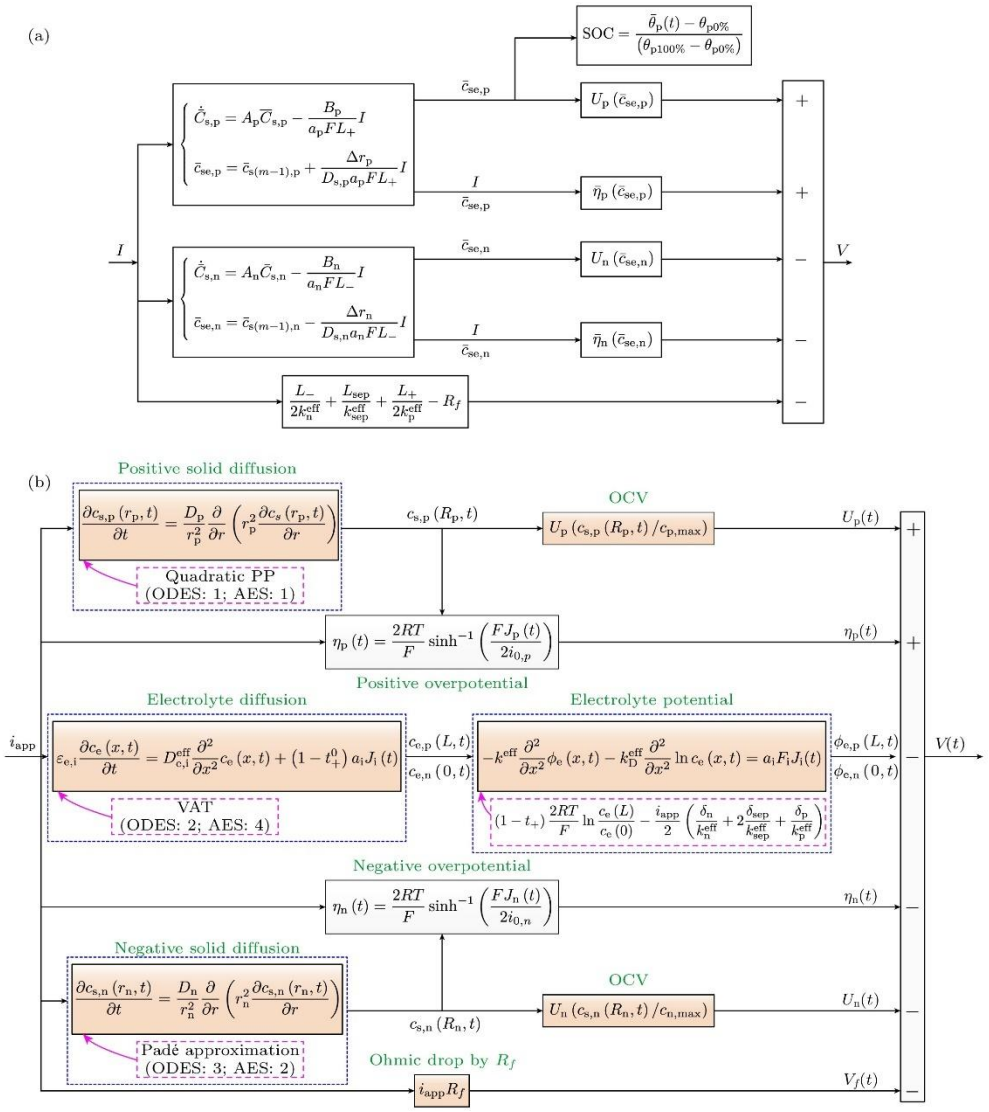
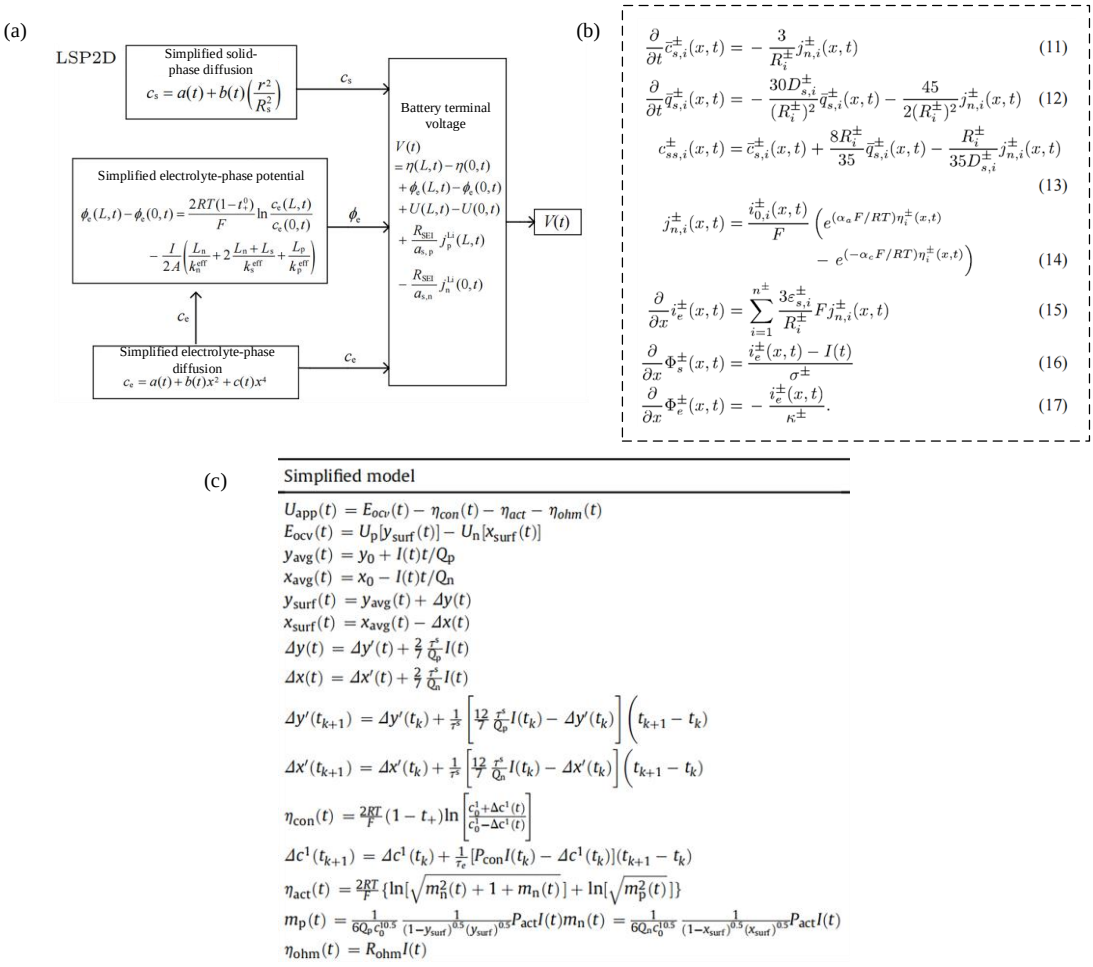


Figure B4. Electrochemical average value model[47,48]



(c)

Simplified model

$$U_{app}(t) = E_{ocv}(t) - \eta_{con}(t) - \eta_{act} - \eta_{ohm}(t)$$
$$E_{ocv}(t) = U_p[y_{surf}(t)] - U_n[x_{surf}(t)]$$
$$y_{avg}(t) = y_0 + I(t)t/Q_p$$
$$x_{avg}(t) = x_0 - I(t)t/Q_n$$
$$y_{surf}(t) = y_{avg}(t) + \Delta y(t)$$
$$x_{surf}(t) = x_{avg}(t) - \Delta x(t)$$
$$\Delta y(t) = \Delta y'(t) + \frac{2}{7}\frac{\tau^s}{Q_p} I(t)$$
$$\Delta x(t) = \Delta x'(t) + \frac{2}{7}\frac{\tau^s}{Q_n} I(t)$$
$$\Delta y'(t_{k+1}) = \Delta y'(t_k) + \frac{1}{\tau^s}\left[\frac{12}{7}\frac{\tau^s}{Q_p} I(t_k) - \Delta y'(t_k)\right]\left(t_{k+1} - t_k\right)$$
$$\Delta x'(t_{k+1}) = \Delta x'(t_k) + \frac{1}{\tau^s}\left[\frac{12}{7}\frac{\tau^s}{Q_n} I(t_k) - \Delta x'(t_k)\right]\left(t_{k+1} - t_k\right)$$
$$\eta_{con}(t) = \frac{2RT}{F}(1 - t_+)\ln\left[\frac{c_0^1 + \Delta c^1(t)}{c_0^1 - \Delta c^1(t)}\right]$$
$$\Delta c^1(t_{k+1}) = \Delta c^1(t_k) + \frac{1}{\tau_e}[P_{con} I(t_k) - \Delta c^1(t_k)](t_{k+1} - t_k)$$
$$\eta_{act}(t) = \frac{2RT}{F}\{\ln[\sqrt{m_n^2(t) + 1} + m_n(t)] + \ln[\sqrt{m_p^2(t)}]\}$$
$$m_p(t) = \frac{1}{6Q_p c_0^{0.5}} \frac{1}{(1 - y_{surf})^{0.5}(y_{surf})^{0.5}} P_{act} I(t) m_n(t) = \frac{1}{6Q_n c_0^{0.5}} \frac{1}{(1 - x_{surf})^{0.5}(x_{surf})^{0.5}} P_{act} I(t)$$
$$\eta_{ohm}(t) = R_{ohm} I(t)$$

Figure B5. LSP2D model[53.54,61–63]

negative electrode	separator	positive electrode
L_n	L_{se}	L_p

Figure B6. One-dimensional model[68]

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