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长椭圓旋轉介質球中的半波振子天綫*

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摘 要

本文从麦克斯韦方程出发，求出有損耗的長椭圓旋轉介質球中一个同焦点的小的長椭圓旋轉形半波振子发射天綫上的电流分布和其輸入阻抗，以及它在有損耗的介質球中和外部自由空間中所产生的場.

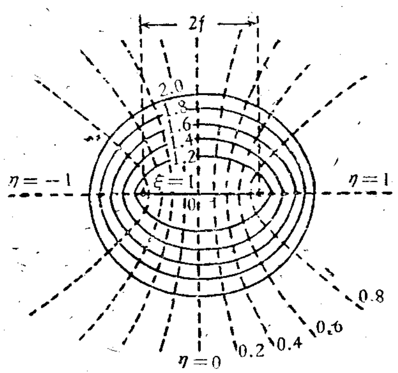
I. 問題的結构及其解

設 ξ, η, ϕ 为長椭圓旋轉坐标系中的三个坐标，如图(1)所示，图中只表示了長椭圓旋轉体的 ϕ 为一个常数的一个断面. 所研究的天綫是一个中間对称饋电的由理想导体制成的長椭圓旋轉形天綫，放在一个同焦点的、有高介質常数或高磁导率或兼而有之的、并且有小損耗的長椭圓旋轉体之中，如图(2)所示. 設 $\mathbf{i}_\xi, \mathbf{i}_\eta, \mathbf{i}_\phi$ 为沿 ξ, η, ϕ 方向的单位矢量，則

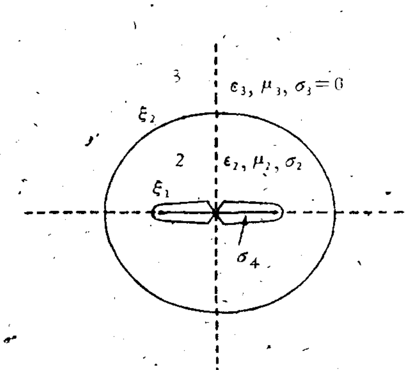
$$\nabla\xi = \frac{1}{h_\xi}\mathbf{i}_\xi, \qquad \nabla\eta = \frac{1}{h_\eta}\mathbf{i}_\eta, \qquad \nabla\phi = \frac{1}{h_\phi}\mathbf{i}_\phi, \tag{1}$$

其中 h_ξ, h_η, h_ϕ 为正交曲面坐标系的尺度系数，其値为

$$h_\xi = f\left(\frac{\xi^2-\eta^2}{\xi^2-1}\right)^{\frac{1}{2}}, \qquad h_\eta = f\left(\frac{\xi^2-\eta^2}{1-\eta^2}\right)^{\frac{1}{2}}, \qquad h_\phi = f[(\xi^2-1)(1-\eta^2)]^{\frac{1}{2}}. \tag{2}$$



$-1 \leqslant \eta \leqslant 1,\ 1 \leqslant \xi < \infty,\ 0 < \phi < 2\pi$
長軸 $a = f\xi$, 短軸 $b = f\sqrt{\xi^2-1}$.
图 1



$\sigma_1 = \infty$
图 2

在正弦情况下，介質2和3中的麦克斯韦方程可以写成:

* 1960年9月8日收到.

$$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon)\mathbf{E}, \quad \nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}. \tag{3}$$

在正交曲面坐标下，它們的形式是：

$$\frac{1}{h_\eta h_\phi}\left[\frac{\partial}{\partial\eta}(h_\phi H_\phi) - \frac{\partial}{\partial\phi}(h_\eta H_\eta)\right]\mathbf{i}_\xi + \frac{1}{h_\phi h_\xi}\left[\frac{\partial}{\partial\phi}(h_\xi H_\xi) - \frac{\partial}{\partial\xi}(h_\phi H_\phi)\right]\mathbf{i}_\eta + \\ + \frac{1}{h_\xi h_\eta}\left[\frac{\partial}{\partial\xi}(h_\eta H_\eta) - \frac{\partial}{\partial\eta}(h_\xi H_\xi)\right]\mathbf{i}_\phi = (\sigma + j\omega\epsilon)(\mathbf{i}_\xi E_\xi + \mathbf{i}_\eta E_\eta + \mathbf{i}_\phi E_\phi), \tag{4}$$

$$\frac{1}{h_\eta h_\phi}\left[\frac{\partial}{\partial\eta}(h_\phi E_\phi) - \frac{\partial}{\partial\phi}(h_\eta E_\eta)\right]\mathbf{i}_\xi + \frac{1}{h_\phi h_\xi}\left[\frac{\partial}{\partial\phi}(h_\xi E_\xi) - \frac{\partial}{\partial\xi}(h_\phi E_\phi)\right]\mathbf{i}_\eta + \\ + \frac{1}{h_\xi h_\eta}\left[\frac{\partial}{\partial\xi}(h_\eta E_\eta) - \frac{\partial}{\partial\eta}(h_\xi E_\xi)\right]\mathbf{i}_\phi = -j\omega\mu(\mathbf{i}_\xi H_\xi + \mathbf{i}_\eta H_\eta + \mathbf{i}_\phi H_\phi). \tag{5}$$

假設場的激发是对 ϕ 对称的，卽外加交变电場对长橢圓旋轉体的长軸是对称的，并且与长軸平行，則我們有

$$E_\phi = 0, \quad \frac{\partial}{\partial\phi} = 0. \tag{6}$$

这时，麦克斯韦方程(4)和(5)变为

$$\left.\begin{aligned} (\sigma + j\omega\epsilon)E_\xi &= \frac{1}{h_\eta h_\phi}\frac{\partial}{\partial\eta}(h_\phi H_\phi) \\ (\sigma + j\omega\epsilon)E_\eta &= -\frac{1}{h_\phi h_\xi}\frac{\partial}{\partial\xi}(h_\phi H_\phi) \\ j\omega\mu H_\phi &= \frac{1}{h_\xi h_\eta}\left[\frac{\partial}{\partial\eta}(h_\xi E_\xi) - \frac{\partial}{\partial\xi}(h_\eta E_\eta)\right] \end{aligned}\right\} \tag{7}$$

在介質 2 中，$\sigma = \sigma_2$，$\mu = \mu_2$，$\epsilon = \epsilon_2$；在介質 3 中，$\sigma_3 = 0$，$\mu = \mu_3$，$\epsilon = \epsilon_3$.

設 $\mathscr{R} = h_\phi H_\phi$，則(7)式可以写成：

$$\left.\begin{aligned} E_\xi &= \frac{1}{(\sigma + j\omega\epsilon)h_\eta h_\phi}\frac{\partial\mathscr{R}}{\partial\eta}, \\ E_\eta &= -\frac{1}{(\sigma + j\omega\epsilon)h_\phi h_\xi}\frac{\partial\mathscr{R}}{\partial\xi}, \\ H_\phi &= \frac{1}{h_\phi}\mathscr{R}. \end{aligned}\right\} \tag{8}$$

将式(8)代入式(7)中的第三式，得

$$(\xi^2 - 1)\frac{\partial^2\mathscr{R}}{\partial\xi^2} + (1 - \eta^2)\frac{\partial^2\mathscr{R}}{\partial\eta^2} + f^2k^2(\xi^2 - \eta^2)\mathscr{R} = 0, \tag{9}$$

其中

$$k^2 = -j\omega\mu(\sigma + j\omega\epsilon). \tag{10}$$

利用分离变量法，設

$$\mathscr{R} = \mathscr{R}_\xi(\xi)\mathscr{R}_\eta(\eta), \tag{11}$$

代入(9)得常微分方程如下：

$$(\xi^2 - 1)\frac{d^2\mathscr{R}_\xi}{d\xi^2} + (f^2k^2\xi^2 - c)\mathscr{R}_\xi = 0, \tag{12}$$

$$(1-\eta^2)\frac{d^2\mathscr{R}_\eta}{d\eta^2}+(c'-f^2k^2\eta^2)\mathscr{R}_\eta=0, \tag{13}$$

其中 c 为分离常数或爱根値．由以上二式，可見 $\mathscr{R}_\xi$ 和 $\mathscr{R}_\eta$ 满足同一方程．作簡单的变換：

$$\mathscr{R}_\xi(\xi)=(\xi^2-1)U(\xi), \tag{14}$$

$$\mathscr{R}_\eta(\eta)=(1-\eta^2)V(\eta); \tag{15}$$

代入(12)和(13)式得：

$$(\xi^2-1)\frac{d^2U}{d\xi^2}+4\xi\frac{dU}{d\xi}+(k^2f^2\xi^2-c+2)U=0, \tag{16}$$

$$(1-\eta^2)\frac{d^2V}{d\eta^2}-4\eta\frac{dV}{d\eta}+(c-2-k^2f^2\eta^2)V=0. \tag{17}$$

此二式与 J. A. Stratton[1,3] 和 P. M. Morse[2] 所得的椭圓旋轉型波函数方程相符合，其中 U 称为輻径函数，V 称为角函数．

在长椭圓旋轉体半波振子天綫的二端，其坐标为 $\xi=\xi_1$，$\eta=\pm1$，电流必須等于零，因此，

$$h_\phi V(\pm1)=0,\text{ 或 } V(\pm1)=\text{有限值}; \tag{18}$$

方程式(13)中的分离常数 c 应由此式来决定．設 $b_l=c-2$，代入(16)和(17)式得：

$$(\xi^2-1)\frac{d^2U}{d\xi^2}+d\xi\frac{dU}{d\xi}-(b_l-k^2f^2\xi^2)U=0, \tag{19}$$

$$(1-\eta^2)\frac{d^2V}{d\eta^2}-4\eta\frac{dV}{d\eta}+(b_l-k^2f^2\eta^2)V=0. \tag{20}$$

以上二式是以下的一个公共标准式的一个特例 $(a=1)$[3]：

$$(1-z^2)\frac{d^2W}{dz^2}-2(a+1)z\frac{dW}{dz}+(b_l-f^2k^2z^2)W=0. \tag{21}$$

对于角函数 V，我們在其解中[3]取第一类椭圓旋轉型波函数，以保証在极点($\eta=\pm1$)处函数为有限值；因此，

$$V(\eta)=S^1_{e1,l}(f,k,\eta)=(\eta^2-1)^{-\frac{1}{2}}\sum_n{}'\,a^l_n j^{n-l}\frac{n!}{(n+2)!}P^1_{n+1}(\eta), \tag{22}$$

其中 P^1_{n+1} 为副拉襄德函数：

$$P^1_{n+1}(\eta)\doteq(1-\eta^2)^{\frac{1}{2}}\frac{d}{d\eta}P_{n+1}(\eta)=(1-\eta^2)^{\frac{1}{2}}\frac{d}{d\eta}\frac{1}{2^{n+1}(n+1)!}\frac{d^{n+1}}{d\eta^{n+1}}(\eta^2-1)^{n+1}, \tag{23}$$

这里 Σ 上的一撇表示当 l 为偶数时 n 取偶整数 $0,2,4,\cdots$，而当 l 为奇整数时 n 則取奇整数 $1,3,5,\cdots$．

为了保証在无穷远处 $(\xi\to\infty)$ 函数有正常特性，我們选取輻径函数 $U(\xi)$ 为第四类椭圓旋轉型波函数：

$$U(\xi)=\mathrm{Re}^4_{1,l}(f,k,\xi)=\frac{1}{fk\xi\sum_n{}'\,a^l_n j^{n-l}}\sum_n{}'\,a^l_n h^{(2)}_{n+1}(f,k,\xi), \tag{24}$$

其中 $h^{(2)}_{n+1}$ 是球型汉克尔函数:

$$h^{(2)}_{n+1}(fk\xi)=\left(\frac{\pi}{2fk\xi}\right)^{\frac{1}{2}}H^{(2)}_{n+3/2}(f,k,\xi), \tag{25}$$

其中 $H^{(2)}_{n+3/2}$ 是半阶的普通汉克尔函数.

因此,場量可以表示为

$$\left.\begin{aligned}
H_\phi &= \sum_l A_l h_\phi U(\xi)V(\eta)e^{j\omega t}=f[(\xi^2-1)(1-\eta^2)]^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_l\,\mathrm{Re}^4_{1,l}(\xi)\mathrm{Se}^1_{1,l}(\eta)\\
E_\xi &= \frac{1}{\sigma+j\omega\epsilon}\left(\frac{\xi^2-1}{\xi^2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_l\,\mathrm{Re}^4_{1,l}(\xi)\frac{d}{d\eta}[(1-\eta^2)\mathrm{Se}^1_{1,l}(\eta)]\\
E_\eta &= -\frac{1}{\sigma+j\omega\epsilon}\left(\frac{1-\eta^2}{\xi^2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_l\,\mathrm{Se}^1_{1,l}(\eta)\frac{d}{d\xi}[(\xi^2-1)\mathrm{Re}^4_{1,l}(\xi)]
\end{aligned}\right\} \tag{26}$$

根据边界条件:

$$\text{在 }\xi=\xi_1,\text{ 即在天綫表面上,}\quad E_{2\eta i}+E_{2\eta r}+E_{a\eta}=0; \tag{27}$$

$$\text{在 }\xi=\xi_2,\text{ 即在介質球表面上,}\quad E_{2\eta i}+E_{2\eta r}=E_{3\eta t}, \tag{28}$$

$$H_{2\phi i}+H_{2\phi r}=H_{3\phi t}; \tag{29}$$

式中脚标 i 代表入射場,r 代表反射場,t 代表传輸場(这都是对介質球面来說的). 式(27)—(29)中的每項可从以下各式写出:

$$\left.\begin{aligned}
E_{2\eta i} &= -\frac{1}{\sigma_2+j\omega\epsilon_2}\left(\frac{1-\eta^2}{\xi^2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{il}\mathrm{Se}^1_{1,l}(f,k_2,\eta)\frac{d}{d\xi}[(\xi^2-1)\mathrm{Re}^4_{1,l}(f,k_2,\xi)],\\
E_{2\eta r} &= -\frac{1}{\sigma_2+j\omega\epsilon_2}\left(\frac{1-\eta^2}{\xi^2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{rl}\mathrm{Se}^1_{1,l}(f,k_2,\eta)\frac{d}{d\xi}[(\xi^2-1)\mathrm{Re}^1_{1,l}(f,k_2,\xi)]\\
E_{3\eta t} &= -\frac{1}{j\omega\epsilon_3}\left(\frac{1-\eta^2}{\xi^2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{tl}\mathrm{Se}^1_{1,l}(f,k_3,\eta)\frac{d}{d\xi}[(\xi^2-1)\mathrm{Re}^4_{1,l}(f,k_3,\xi)],\\
H_{2\phi i} &= f[(\xi^2-1)(1-\eta^2)]^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{il}\,\mathrm{Re}^4_{1,l}(f,k_2,\xi)\mathrm{Se}^1_{1,l}(f,k_2,\eta),\\
H_{2\phi r} &= f[(\xi^2-1)(1-\eta^2)]^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{rl}\,\mathrm{Re}^1_{1,l}(f,k_2,\xi)\mathrm{Se}^1_{1,l}(f,k_2,\eta),\\
H_{3\phi t} &= f[(\xi^2-1)(1-\eta^2)]^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{tl}\,\mathrm{Re}^4_{1,l}(f,k_3,\xi)\mathrm{Se}^1_{1,l}(f,k_3,\eta).
\end{aligned}\right\} \tag{30}$$

設在椭圓旋轉面 $\xi=\xi_1$ 上的外加場可以展开如下:

$$E_{a\eta}=-\frac{1}{\sigma_2+j\omega\epsilon_2}\left(\frac{1-\eta^2}{\xi^2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}B_l\,\mathrm{Se}^1_{1,l}(f,k_2,\eta). \tag{31}$$

将(30)和(31)式代入(27)—(29)式中,則得

$$-\frac{1}{\sigma_2+j\omega\epsilon_2}\left(\frac{1-\eta^2}{\xi_1^2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{il}\mathrm{Se}^1_{1,l}(f,k_2,\eta)\frac{d}{d\xi_1}[(\xi_1^2-1)\mathrm{Re}^4_{1,l}(f,k_2,\xi_1)]$$

$$-\frac{1}{\sigma_2+j\omega\epsilon_2}\left(\frac{1-\eta^2}{\xi_1^2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{rl}\mathrm{Se}^1_{1,l}(f,k_2,\eta)\frac{d}{d\xi_1}[(\xi_1^2-1)\mathrm{Re}^1_{1,l}(f,k_2,\xi_1)]$$

$$-\frac{1}{\sigma_2+j\omega\epsilon_2}\left(\frac{1-\eta^2}{\xi_1^2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}B_l\,\mathrm{Se}_{1,l}^{1}(f,k_2,\eta)=0, \tag{32}$$

$$-\frac{1}{\sigma_2+j\omega\epsilon_2}\left(\frac{1-\eta^2}{\xi_2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{il}\mathrm{Se}_{1,l}^{1}(f,k_2,\eta)\frac{d}{d\xi_2}[(\xi_2^2-1)\mathrm{Re}_{1,l}^{4}(f,k_2,\xi_2)]-$$
$$-\frac{1}{\sigma_2+j\omega\epsilon_2}\left(\frac{1-\eta^2}{\xi_2^2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{rl}\mathrm{Se}_{1,l}^{1}(f,k_2,\eta)\frac{d}{d\xi_2}[(\xi_2^2-1)\mathrm{Re}_{1,l}^{1}(f,k_2,\xi_2)]=$$
$$=-\frac{1}{j\omega\epsilon_3}\left(\frac{1-\eta^2}{\xi_2^2-\eta^2}\right)^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{tl}\mathrm{Se}_{1,l}^{1}(f,k_3,\eta)\frac{d}{d\xi_2}[(\xi_2^2-1)\mathrm{Re}_{1,l}^{4}(f,k_3,\xi_2)], \tag{33}$$

$$f[(\xi_2^2-1)(1-\eta^2)]^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{il}\,\mathrm{Re}_{1,l}^{4}(f,k_2,\xi_2)\mathrm{Se}_{1,l}^{1}(f,k_2,\eta)+$$
$$+f[(\xi_2^2-1)(1-\eta^2)]^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{rl}\,\mathrm{Re}_{1,l}^{1}(f,k_2,\xi_2)\mathrm{Se}_{1,l}^{1}(f,k_2,\eta)=$$
$$=f[(\xi^2-1)(1-\eta^2)]^{\frac{1}{2}}e^{j\omega t}\sum_{l=0}^{\infty}A_{tl}\,\mathrm{Re}_{1,l}^{4}(f,k_3,\xi_2)\mathrm{Se}_{1,l}^{1}(f,k_3,\eta). \tag{34}$$

在以上各式中，对于反射場中的輻径波函数，我們取第一类輻径波函数 $\mathrm{Re}_{1,l}^{1}(f,k,\xi)$ 而不用第四类，以保証在极点上函数为有限值[3]：

$$\mathrm{Re}_{1,l}^{1}(f,k,\xi)=\frac{1}{\xi\sum_n{}'\,a_n^l\,j^{n-l}}\sum_n{}'\,a_n^l\sqrt{\frac{\pi}{2fk\xi}}J_{n+3/2}(f,k,\xi). \tag{35}$$

解(31)—(33)式得出常数 A_{il}, A_{rl} 和 A_{tl} 如下：

$$A_{il}=\frac{b'[\mathrm{Re}_{1,l}^{1}(f,k_2,\xi_2)-de]}{\mathrm{Re}_{1,l}^{4}(f,k_2,\xi_2)+\mathrm{Re}_{1,l}^{1}(f,k_2,\xi_2)-ec-ed}, \tag{36}$$

$$A_{rl}=b'-A_{il}, \tag{37}$$

$$A_{tl}=\frac{1}{\mathrm{Re}_{1,l}^{4}(f,k_3,\xi_2)}[A_{il}\,\mathrm{Re}_{1,l}^{4}(f,k_2,\xi_2)+A_{rl}\,\mathrm{Re}_{1,l}^{1}(f,k_2,\xi_2)], \tag{38}$$

其中

$$\left.\begin{aligned}
b'&=\frac{B_l}{\dfrac{d}{d\xi_1}\{(\xi_1^2-1)[\mathrm{Re}_{1,l}^{4}(f,k_2\xi_1)-\mathrm{Re}_{1,l}^{1}(f,k_2,\xi_1)]\}},\\
c&=\frac{d}{d\xi_2}[\xi_2^2-1)\mathrm{Re}_{1,l}^{4}(f,k_2,\xi_2)],\\
d&=\frac{d}{d\xi_2}[(\xi_2^2-1)\mathrm{Re}_{1,l}^{1}(f,k_2,\xi_2)],\\
e&=\frac{j\omega\epsilon_3}{\sigma_2+j\omega\epsilon_2}\,\frac{\mathrm{Re}_{1,l}^{4}(f,k_3,\xi_2)}{\dfrac{d}{d\xi_2}[(\xi_2^2-1)\mathrm{Re}_{1,l}^{4}(f,k_3,\xi_2)]};
\end{aligned}\right\} \tag{39}$$

其中

$$k_2^2=-j\omega\mu_2(\sigma_2+j\omega\epsilon_2),\qquad k_3^2=\omega^2\epsilon_3\mu_3 \tag{40}$$

假設在橢圓旋轉球面 $\xi=\xi_1$ 的赤道上 $(\eta=0)$ 加一对称电压于縫隙的長度 $h_\eta\Delta\eta$

上，則

$$\text{当 } |\eta| < \Delta\eta/2, \qquad E_{a\eta} = -V/h_\eta \Delta\eta,$$
$$\text{当 } |\eta| > \Delta\eta/2, \qquad E_{a\eta} = 0.$$

函数 $\mathrm{Se}^1_{1,l}(\eta)$ 具有正交性，即当 $l \neq m$ 时，

$$\int_{-1}^{1} (1-\eta^2) V_l V_m \, d\eta = 0. \tag{41}$$

其証明如下：設 $\mathscr{R}_l$ 和 $\mathscr{R}_m$ 都是方程(13)的解，則得

$$(1-\eta^2)\frac{d^2\mathscr{R}_l}{d\eta^2} + (c_l - f^2k^2\eta^2)\mathscr{R}_l = 0, \tag{42}$$

$$(1-\eta^2)\frac{d^2\mathscr{R}_m}{d\eta^2} + (c_m - f^2k^2\eta^2)\mathscr{R}_m = 0. \tag{43}$$

用 $\mathscr{R}_m$ 乘(42)式，用 $\mathscr{R}_l$ 乘(43)式，然后相減得：

$$(1-\eta^2)\frac{d}{d\eta}\left(\mathscr{R}_l\frac{d\mathscr{R}_m}{d\eta} - \mathscr{R}_m\frac{d\mathscr{R}_l}{d\eta}\right) + (c_m - c_l)\mathscr{R}_l\mathscr{R}_m = 0. \tag{44}$$

将(44)式除以 $1-\eta^2$ 并从 -1 到 $+1$ 之間对 η 积分得：

$$(c_m - c_l)\int_{-1}^{+1}(1-\eta^2)^{-1}\mathscr{R}_l\mathscr{R}_m \, d\eta = \left(\mathscr{R}_l\frac{d\mathscr{R}_m}{d\eta} - \mathscr{R}_m\frac{d\mathscr{R}_l}{d\eta}\right)\Bigg|_{-1}^{+1} = 0 \tag{45}$$

这是因为 $\mathscr{R} = (1-\eta^2)V$，而当 $\eta = \pm 1$ 时由于已选为第一类椭圓旋轉函数从而为有限值，所以 $\mathscr{R}(\pm 1) = 0$. 在 $l \neq m$，或 $c_m \neq c_l$ 情况下，則

$$\int_{-1}^{+1}(1-\eta^2)^{-1}\mathscr{R}_l\mathscr{R}_m \, d\eta = 0. \tag{46}$$

又因为 $\mathscr{R} = (1-\eta^2)V$，所以

$$\int_{-1}^{+1}(1-\eta^2)V_l V_m \, d\eta = 0.$$

这就是所要証明的(40)式. 由于这种正交性，我們可以求出系数 B_l. 将(32)式两边都乘以 $-V_l(\sigma_2 + j\omega\epsilon_2)[(\xi^2-\eta^2)(1-\eta^2)]^{\frac{1}{2}}$ 并弃去 $e^{j\omega t}$ 而取其复数值，然后从 $\eta = -1$ 到 $\eta = +1$ 积分，得

$$-\int_{-1}^{+1}(\sigma_2 + j\omega\epsilon_2)[(\xi_1^2-\eta^2)(1-\eta^2)]^{\frac{1}{2}}E_{a\eta}V_l \, d\eta = \int_{-1}^{+1}(1-\eta^2)V_l\sum_{l=0}^{\infty}B_l V_l \, d\eta =$$
$$= B_l\int_{-1}^{+1}(1-\eta^2)V_l^2 \, d\eta, \tag{47}$$

$$\therefore \quad B_l = \frac{-(\sigma_2 + j\omega\epsilon_2)}{N_l}\int_{-1}^{+1}[(\xi_1^2-\eta^2)(1-\eta^2)]^{\frac{1}{2}}E_{a\eta}V_l \, d\eta, \tag{48}$$

式中 N_l 为归正系数：

$$N_l = \int_{-1}^{+1}(1-\eta^2)[\mathrm{Se}^1_{1,l}(f, k_2, \eta)]^2 \, d\eta. \tag{49}$$

因为当 $|\eta| > \Delta\eta/2$ 时，$E_{a\eta} = 0$，所以

$$B_l = \frac{\sigma_2 + j\omega\epsilon_2}{N_l}\int_{-\Delta\eta/2}^{\Delta\eta/2}[(\xi_1^2-\eta^2)(1-\eta^2)]^{\frac{1}{2}}\frac{V}{h_\eta\Delta\eta}\mathrm{Se}^1_{1,l}(f, k_2, 0)\, d\eta =$$
$$= \frac{\sigma_2 + j\omega\epsilon_2}{N_l}\frac{V}{f}\mathrm{Se}^1_{1,l}(f, k_2, 0). \tag{50}$$

如設 $\sigma_2 = 0$, $\epsilon_2 = \epsilon_3 = \epsilon_0$, $\mu_2 = \mu_3 = \mu_0$, 則此式与 Chu 和 Stratton[4] 所得的式子相符.

II. 天綫上的电流分布和輸入阻抗

由于对称性,天綫电流等于:

$$I(\eta) = 2\pi h_\phi H_{\phi t} = 2\pi h_\phi (H_{\phi i} + H_{\phi r})|_{\xi=\xi_1} =$$
$$= 2\pi f^2(\xi_1^2 - 1)(1-\eta^2)\sum_{l=0}^{\infty} \mathrm{Se}_{1,l}^1(f, k_2, \eta)[A_{il}\,\mathrm{Re}_{1,l}^4(f, k_2, \xi_1) +$$
$$+ A_{rl}\,\mathrm{Re}_{1,l}^1(f, k_2, \xi_1)]. \tag{51}$$

假設外加电压 V 是用一双錐形传輸綫饋給的(图 2),縫隙的长度为 S, 則

$$dS_\eta = \frac{f\sqrt{\xi_1^2-\eta^2}}{\sqrt{1-\eta^2}}\,d\eta \tag{52}$$

在赤道面上, $\eta = 0$; 因此在赤道面附近可以近似地得:

$$dS_\eta = f\xi_1\,d\eta \tag{53}$$

縫隙是从 $\eta = -\dfrac{S}{2f\xi_1}$ 到 $\eta = \dfrac{S}{2f\xi_1}$. 如果电压是均匀分布的,則在 $\eta = \dfrac{s}{2f\xi_1}$ 处的电流为

$$I\left(\frac{1}{2}S\right) = \sum_{l=0}^{\infty} 2\pi f^2(\xi_1^2-1)[A_{il}\,\mathrm{Re}_{1,l}^4(f, k_2, \xi_1) +$$
$$+ A_{rl}\,\mathrm{Re}_{1,l}^1(f, k_2, \xi_1)]\frac{1}{S/f\xi_1}\int_{-\frac{s}{2f\xi_1}}^{\frac{s}{2f\xi_1}}(1-\eta^2)\mathrm{Se}_{1,l}^1(f, k_2, \eta)d\eta. \tag{54}$$

因此,天綫的輸入导納为

$$Y_i = \frac{I\left(\frac{1}{2}S\right)}{V} = \sum_{l=0}^{\infty} 2\pi f^2(\xi_1^2-1)\left[\frac{A_{il}}{V}\mathrm{Re}_{1,l}^4(f, k_2, \xi_1) + \frac{A_{rl}}{V}\mathrm{Re}_{1,l}^1(f, k_2, \xi_1)\right]\times$$
$$\times\frac{1}{S/f\xi_1}\int_{-\frac{s}{2f\xi}}^{\frac{s}{2f\xi_1}}(1-\eta^2)\mathrm{Se}_{1,l}^1(f, k_2, \eta)d\eta. \tag{55}$$

設

$$g = \frac{\mathrm{Re}_{1,l}^1(f,k_2,\xi_2) - de}{\mathrm{Re}_{1,l}^4(f,k_2,\xi_2) + \mathrm{Re}_{1,l}^1(f,k_2,\xi_2) - ec - ed} \tag{56}$$

$$P = \frac{1}{d\xi_1}\left\{(\xi_1^2-1)[\mathrm{Re}_{1,l}^4(f,k_2,\xi_1) - \mathrm{Re}_{1,l}^1(f, k_2, \xi_1)]\right\} \tag{57}$$

$$Q = \frac{\sigma_2 + j\omega\epsilon_2}{N_l}\mathrm{Se}_{1,l}^1(f,k_2,0), \tag{58}$$

則

$$Y_i = \sum_{l=0}^{\infty} 2\pi f^2(\xi_1^2-1)\left[\frac{Qg}{P}\mathrm{Re}_{1,l}^4(f, k_2, \xi_1) + \frac{Q}{P}(1-g)\mathrm{Re}_{1,l}^1(f, k_2, \xi_1)\right]\times$$
$$\times\frac{1}{S/f\xi_1}\int_{-\frac{s}{2f\xi_1}}^{\frac{s}{2f\xi_1}}(1-\eta^2)\mathrm{Se}_{1,l}^1(f, k_2, \eta)d\eta. \tag{59}$$

这里我們用了双錐形传輸綫饋电并保持縫隙为有限，而不用 δ- 函数源，以避免电納的級数的发散，因此在导納的式子里出現了縫隙的尺寸 S 作为一个参数。

III. 場量的普遍表达式

在介質 3 中，

$$E_\xi = \frac{1}{j\omega\epsilon_3}\left(\frac{\xi^2-1}{\xi^2-\eta^2}\right)^{\frac{1}{2}} e^{j\omega t}\sum_{l=0}^{\infty} A_{tl}\,\mathrm{Re}^4_{1,l}(f,k_3,\xi)\frac{d}{d\eta}[(1-\eta^2)\mathrm{Se}^1_{1,l}(f,k_3,\eta)], \quad (60)$$

$$E_\eta = -\frac{1}{j\omega\epsilon_3}\left(\frac{1-\eta^2}{\xi^2-\eta^2}\right)^{\frac{1}{2}} e^{j\omega t}\sum_{l=0}^{\infty} A_{tl}\mathrm{Se}^1_{1,l}(f,k_3,\eta)\frac{d}{d\xi}[(\xi^2-1)\mathrm{Re}^4_{1,l}(f,k_3,\xi)], \quad (61)$$

$$H_\phi = f[(\xi^2-1)(1-\eta^2)]^{\frac{1}{2}} e^{j\omega t}\sum_{l=0}^{\infty} A_{tl}\,\mathrm{Re}^4_{1,l}(f,k_3,\xi)\mathrm{Se}^1_{1,l}(f,k_3,\eta). \quad (62)$$

在介質 2 中，

$$E_\xi = \frac{1}{\sigma_2+j\omega\epsilon_2}\left(\frac{\xi^2-1}{\xi^2-\eta^2}\right)^{\frac{1}{2}} e^{j\omega t}\sum_{l=0}^{\infty}\frac{d}{d\eta}[(1-\eta^2)\mathrm{Se}^1_{1,l}(f,k_2,\eta)][A_{il}\,\mathrm{Re}^4_{1,l}(f,k_2,\xi)+$$
$$+A_{rl}\,\mathrm{Re}^1_{1,l}(f,k_2,\xi)], \quad (63)$$

$$E_\eta = -\frac{1}{\sigma_2+j\omega\epsilon_2}\left(\frac{1-\eta^2}{\xi^2-\eta^2}\right)^{\frac{1}{2}} e^{j\omega t}\sum_{l=0}^{\infty}\mathrm{Se}^1_{1,l}(f,k_2,\eta)\left\{A_{il}\frac{d}{d\xi}[(\xi^2-1)\mathrm{Re}^4_{1,l}(f,k_2,\xi)]+\right.$$
$$\left.+A_{rl}\frac{d}{d\xi}[(\xi^2-1)\mathrm{Re}^1_{1,l}(f,k_2,\xi)]\right\} \quad (64)$$

$$H_\phi = f[(\xi^2-1)(1-\eta^2)]^{\frac{1}{2}} e^{j\omega t}\sum_{l=0}^{\infty}\mathrm{Se}^1_{1,l}(f,k_2,\eta)[A_{il}\,\mathrm{Re}^4_{1,l}(f,k_2,\xi)+$$
$$+A_{rl}\,\mathrm{Re}^1_{1,l}(f,k_2,\xi)]. \quad (65)$$

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A DIPOLE ANTENNA EMBEDDED IN A PROLATE DIELECTRIC SPHEROID

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ABSTRACT

A prolate spheroidal dipole antenna embedded in a larger dielectric confocal prolate spheroid with finite conductivity is analized as a boundary value problem in electromagnetic theory; the expressions for the fields, antenna current distribution and input impedance are obtained.