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三自由度二阶非线性耦合动力学系统守恒量的扩展 Prelle-Singer 求法

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用扩展 Prelle-Singer 法(扩展 P-S 法)求三自由度二阶非线性耦合动力学系统的守恒量,得到了 6 个积分乘子满足的确定方程、约束方程和守恒量的一般形式,并讨论了确定积分乘子的方法. 最后,用扩展 P-S 法求得了三质点 Tada 晶格问题的两个守恒量.

关键词: 扩展 Prelle-Singer 法, 三自由度非线性耦合动力学系统, 守恒量

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1. 引 言

寻求力学系统的守恒量一直是力学、物理学、数学研究者关注的问题,长期以来,寻找力学系统的守恒量有多种方法,如 Noether 对称性法[1—5]、Lie 对称性法[5—7]、Mei 对称性法[8—10]、Ermakov 方法[11—13]、Poisson 括号法[14—17]、直接积分法[18—22]. 用 Noether 对称性法、Lie 对称性法和 Mei 对称性法求守恒量都要用到群的无限小变换,理论性强且比较抽象. Ermakov 方法只能求可表示成 Ermakov 形式的二自由度系统的守恒量,而对三自由度系统无能为力. Poisson 括号法只能求线性耦合系统的守恒量. 直接积分法用到一阶微分式比较多,且不相互独立,导致用到的积分乘子也比较多. 近几年来,一些数学家致力于研究用 Prelle-Singer 法(P-S 法)求微分方程的第一积分(守恒量)[23—28]. 1983 年, Prelle 和 Singer[23] 提出了一种根据组成解的基本初等函数求得一阶微分方程解的有效方法,即 P-S 法,其优点是只要一阶微分方程存在基本初等函数组成的解,则 P-S 法一定能找到其解和第一积分. P-S 法求第一积分的基本思路是先假设系统存在第一积分 I(守恒量),然后用若干个积分乘子(未知函数)去乘以恒为零的一阶微分式,通过比较系数法求得积分乘子,从而求得第一积分(守恒量). Guha 等[24] 将 P-S 法进行扩展(扩展 P-S 法)并应用于求解二阶及二阶以上的相互耦合的微分方程组的第一积分. 而力学系统中出现的多为二阶非线性耦合的微分方程,因此,扩展 P-S 法可以用于求一般力学系统的守恒量,其理论简洁直观,便于掌握和应用.

我们知道,许多实际力学系统的运动微分方程往往是三自由度二阶非线性耦合方程,形如 $\ddot{q}_i = \phi_i(q_j)$ $(i,j = 1,2,3)$(不显含时间和广义速度),如耦合谐振子、三质点 Tada 晶格等问题. 本文将扩展 P-S 法应用于求三自由度二阶非线性耦合动力学系统的守恒量,得到 6 个积分乘子满足的确定方程、约束方程和守恒量的一般形式,并讨论了确定积分乘子的方法. 最后用扩展 P-S 法求得了三质点 Tada 晶格问题的两个守恒量.

随着自由度的增加,计算量会大增,关于二阶力学系统守恒量的扩展 P-S 法的应用研究目前还局限于二自由度系统. 因此,本文将扩展 P-S 法应用于求三自由度二阶非线性耦合自治动力学系统的守恒量有一定的实际意义和推广价值.

2. 用扩展 P-S 法求三自由度系统守恒量的基本理论

三自由度非线性耦合动力学系统的运动微分方程可表示为

$$\ddot{q}_i = \phi_i(q_j) \qquad (i,j = 1,2,3). \tag{1}$$

显然

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$$\frac{\partial \phi_i}{\partial t} = 0,$$

$$\frac{\partial \phi_i}{\partial \dot{q}_j} = 0,$$

其中 $i,j=1,2,3$. 设此力学系统存在守恒量

$$I = I(t, q_i, \dot{q}_i) \qquad (i = 1,2,3),$$

则

$$\mathrm{d}I = I_t \mathrm{d}t + \sum_{i=1}^{3} I_{q_i} dq_i + \sum_{i=1}^{3} I_{\dot{q}_i} \mathrm{d}\dot{q}_i. \qquad (2)$$

这里的 $I_t, I_{q_i}, I_{\dot{q}_i}$ 分别表示 I 对 $t, q_i, \dot{q}_i$ 的偏导数. 利用 6 个独立且恒为零的一阶微分式 $\mathrm{d}q_i - \dot{q}_i \mathrm{d}t, \mathrm{d}\dot{q}_i - \phi_i \mathrm{d}t$ $(i = 1,2,3)$, 可得以下 3 个恒为零的公式:

$$\mathrm{d}\dot{q}_1 - \phi_1 \mathrm{d}t + \sum_{i=1}^{3} R_i(\mathrm{d}q_i - \dot{q}_i \mathrm{d}t) = 0, \qquad (3a)$$

$$\mathrm{d}\dot{q}_2 - \phi_2 \mathrm{d}t + \sum_{i=1}^{3} S_i(\mathrm{d}q_i - \dot{q}_i \mathrm{d}t) = 0, \qquad (3b)$$

$$\mathrm{d}\dot{q}_3 - \phi_3 \mathrm{d}t + \sum_{i=1}^{3} T_i(\mathrm{d}q_i - \dot{q}_i \mathrm{d}t) = 0, \qquad (3c)$$

式中

$$R_i = R_i(t, q_j, \dot{q}_j),$$

$$S_i = S_i(t, q_j, \dot{q}_j),$$

$$T_i = T_i(t, q_j, \dot{q}_j) \qquad (i,j = 1,2,3)$$

为任意函数. 而(2)式与(3)式必成比例[23—28], 因此, 用积分乘子

$$U = U(t, q_i, \dot{q}_i),$$

$$V = V(t, q_i, \dot{q}_i),$$

$$W = W(t, q_i, \dot{q}_i) \qquad (i = 1,2,3)$$

分别乘以(3a), (3b), (3c)式后求和应与(2)式相等, 即

$$\begin{aligned}\mathrm{d}I = &-(U\phi_1 + V\phi_2 + W\phi_3 + K\dot{q}_1 + M\dot{q}_2 + N\dot{q}_3)\mathrm{d}t \\ &+ K\mathrm{d}q_1 + M\mathrm{d}q_2 + N\mathrm{d}q_3 + U\mathrm{d}\dot{q}_1 + V\mathrm{d}\dot{q}_2 + W\mathrm{d}\dot{q}_3 \\ =&0,\end{aligned} \qquad (4)$$

式中

$$K = UR_1 + VS_1 + WT_1,$$

$$M = UR_2 + VS_2 + WT_2,$$

$$N = UR_3 + VS_3 + WT_3.$$

比较(4)式与(2)式中 $\mathrm{d}t, \mathrm{d}q_1, \mathrm{d}q_2, \mathrm{d}q_3, \mathrm{d}\dot{q}_1, d\dot{q}_2, \mathrm{d}\dot{q}_3$ 的系数可得

$$\begin{aligned} I_t &= -(U\phi_1 + V\phi_2 + W\phi_3 + K\dot{q}_1 + M\dot{q}_2 + N\dot{q}_3), \\ I_{q_1} &= K, \\ I_{q_2} &= M, \\ I_{q_3} &= N, \\ I_{\dot{q}_1} &= U, \\ I_{\dot{q}_2} &= V, \\ I_{\dot{q}_3} &= W. \end{aligned} \qquad (5)$$

根据相容条件

$$I_{tq_i} = I_{q_i t},$$

$$I_{t\dot{q}_i} = I_{\dot{q}_i t},$$

$$I_{q_i q_j} = I_{q_j q_i},$$

$$I_{q_i \dot{q}_j} = I_{\dot{q}_j q_i},$$

$$I_{\dot{q}_i \dot{q}_j} = I_{\dot{q}_j \dot{q}_i} \qquad (i = 1,2,3),$$

可得以下 21 个方程:

$$\dot{K} = -U\phi_{1q_1} - V\phi_{2q_1} - W\phi_{3q_1}, \qquad (6a)$$

$$\dot{M} = -U\phi_{1q_2} - V\phi_{2q_2} - W\phi_{3q_2}, \qquad (6b)$$

$$\dot{N} = -U\phi_{1q_3} - V\phi_{2q_3} - W\phi_{3q_3}, \qquad (6c)$$

$$\dot{U} = -K, \qquad (6d)$$

$$\dot{V} = -M, \qquad (6e)$$

$$\dot{W} = -N; \qquad (6f)$$

$$K_{q_2} = M_{q_1}, \quad K_{q_3} = N_{q_1}, \quad M_{q_3} = N_{q_2}, \qquad (6g)$$

$$K_{\dot{q}_1} = U_{q_1}, \quad K_{\dot{q}_2} = V_{q_1}, \quad K_{\dot{q}_3} = W_{q_1}, \qquad (6h)$$

$$M_{\dot{q}_1} = U_{q_2}, \quad M_{\dot{q}_2} = V_{q_2}, \quad M_{\dot{q}_3} = W_{q_2}, \qquad (6i)$$

$$N_{\dot{q}_1} = U_{q_3}, \quad N_{\dot{q}_2} = V_{q_3}, \quad N_{\dot{q}_3} = W_{q_3}, \qquad (6j)$$

$$U_{\dot{q}_2} = V_{\dot{q}_1}, \quad U_{\dot{q}_3} = W_{\dot{q}_1}, \quad V_{\dot{q}_3} = W_{\dot{q}_2}. \qquad (6k)$$

这里(6a)—(6f)式为积分乘子的确定方程, (6g)—(6k)式为积分乘子的约束方程; $\dot{K}$ 表示 K 对时间的全导数, 即

$$\dot{K} = K_t + \sum_{i=1}^{3} K_{q_i}\dot{q}_i + \sum_{i=1}^{3} K_{\dot{q}_i}\phi_i,$$

其他类同.

3. 积分乘子的确定与守恒量的一般表达式

由(6)式可知,要确定 6 个积分乘子,关键是先确定 U,V,W 三个乘子,将它们分别代入(6d),(6e),(6f)式便可得另三个积分乘子 K,M,N, 而且每组解必须满足约束方程(6g)—(6k).

现将(6d),(6e),(6f)式分别对时间求导一次并代入(6a),(6b),(6c)式,可得关于 U,V,W 的二阶耦合变系数微分方程组

$$\ddot{U} = U\phi_{1q_1} + V\phi_{2q_1} + W\phi_{3q_1}, \tag{7a}$$

$$\ddot{V} = U\phi_{1q_2} + V\phi_{2q_2} + W\phi_{3q_2}, \tag{7b}$$

$$\ddot{W} = U\phi_{1q_3} + V\phi_{2q_3} + W\phi_{3q_3}. \tag{7c}$$

事实上,方程组(7)不封闭,无法求得其通解,只能根据研究问题的特征(是否为自治系统、Lagrange 函数的形式等)先假设 U,V,W 的拟解,如自治系统一般可假设 U,V,W 不显含时间 t,形如

$$U = a_1(q_i)\dot{q}_1 + b_1(q_i)\dot{q}_2 + c_1(q_i)\dot{q}_3 + d_1(q_i), \tag{8a}$$

$$V = a_2(q_i)\dot{q}_1 + b_2(q_i)\dot{q}_2 + c_2(q_i)\dot{q}_3 + d_2(q_i), \tag{8b}$$

$$W = a_3(q_i)\dot{q}_1 + b_3(q_i)\dot{q}_2 + c_3(q_i)\dot{q}_3 + d_3(q_i). \tag{8c}$$

将 U,V,W 及 $\ddot{U},\ddot{V},\ddot{W}$ 联合(1)式代入(7)式,并比较等式两边广义速度组合项 $\dot{q}_1^m\dot{q}_2^n\dot{q}_3^l$ $(m,n,l = 0,1,2,3)$ 的系数,可得一组 a_i,b_i,c_i,d_i $(i = 1,2,3)$ 关于 q_1,q_2,q_3 的偏微分方程组

$$\begin{aligned}
&a_{1q_1q_1} = b_{1q_2q_2} = c_{1q_3q_3} = 0,\\
&d_{1q_1q_1} = d_{1q_2q_2} = d_{1q_3q_3} = d_{1q_1q_2} = d_{1q_1q_3} = d_{1q_2q_3} = 0,\\
&2a_{1q_1q_2} + b_{1q_1q_1} = 0,\\
&2a_{1q_1q_3} + c_{1q_1q_1} = 0,\\
&2b_{1q_1q_2} + a_{1q_2q_2} = 0,\\
&2b_{1q_2q_3} + c_{1q_2q_2} = 0,\\
&2c_{1q_1q_3} + a_{1q_3q_3} = 0,\\
&2c_{1q_2q_3} + b_{1q_3q_3} = 0,\\
&a_{1q_2q_3} + b_{1q_1q_3} + c_{1q_1q_2} = 0,\\
&(a_1\phi_1 + b_1\phi_2 + c_1\phi_3)_{q_1} + 2a_{1q_1}\phi_1 + (a_{1q_2} + b_{1q_1})\phi_2 + (a_{1q_3} + c_{1q_1})\phi_3\\
&\quad = a_1\phi_{1q_1} + a_2\phi_{2q_1} + a_3\phi_{3q_1},\\
&(a_1\phi_1 + b_1\phi_2 + c_1\phi_3)_{q_2} + 2b_{1q_2}\phi_2 + (a_{1q_2} + b_{1q_1})\phi_1 + (b_{1q_3} + c_{1q_2})\phi_3\\
&\quad = b_1\phi_{1q_1} + b_2\phi_{2q_1} + b_3\phi_{3q_1},\\
&(a_1\phi_1 + b_1\phi_2 + c_1\phi_3)_{q_3} + 2c_{1q_3}\phi_3 + (a_{1q_3} + c_{1q_1})\phi_1 + (b_{1q_3} + c_{1q_2})\phi_2\\
&\quad = c_1\phi_{1q_1} + c_2\phi_{2q_1} + c_3\phi_{3q_1},\\
&d_{1q_1}\phi_1 + d_{1q_2}\phi_2 + d_{1q_3}\phi_3 = c_1\phi_{1q_1} + c_2\phi_{2q_1} + c_3\phi_{3q_1};
\end{aligned} \tag{9a}$$

$$\begin{aligned}
&a_{2q_1q_1} = b_{2q_2q_2} = c_{2q_3q_3} = 0,\\
&d_{2q_1q_1} = d_{2q_2q_2} = d_{2q_3q_3} = d_{2q_1q_2} = d_{2q_1q_3} = d_{2q_2q_3} = 0,\\
&2a_{2q_1q_2} + b_{2q_1q_1} = 0,\\
&2a_{2q_1q_3} + c_{2q_1q_1} = 0,\\
&2b_{2q_1q_2} + a_{2q_2q_2} = 0,\\
&2b_{2q_2q_3} + c_{2q_2q_2} = 0,\\
&2c_{2q_1q_3} + a_{2q_3q_3} = 0,\\
&2c_{2q_2q_3} + b_{2q_3q_3} = 0,\\
&a_{2q_2q_3} + b_{2q_1q_3} + c_{2q_1q_2} = 0,\\
&(a_2\phi_1 + b_2\phi_2 + c_2\phi_3)_{q_1} + 2a_{2q_1}\phi_1 + (a_{2q_2} + b_{2q_1})\phi_2 + (a_{2q_3} + c_{2q_1})\phi_3\\
&\quad = a_1\phi_{1q_2} + a_2\phi_{2q_2} + a_3\phi_{3q_2},\\
&(a_2\phi_1 + b_2\phi_2 + c_2\phi_3)_{q_2} + 2b_{2q_2}\phi_2 + (a_{2q_2} + b_{2q_1})\phi_1 + (b_{2q_3} + c_{2q_2})\phi_3\\
&\quad = b_1\phi_{1q_2} + b_2\phi_{2q_2} + b_3\phi_{3q_2},\\
&(a_2\phi_1 + b_2\phi_2 + c_2\phi_3)_{q_3} + 2c_{2q_3}\phi_3 + (a_{2q_3} + c_{2q_1})\phi_1 + (b_{2q_3} + c_{2q_2})\phi_2\\
&\quad = c_1\phi_{1q_2} + c_2\phi_{2q_2} + c_3\phi_{3q_2},\\
&d_{2q_1}\phi_1 + d_{2q_2}\phi_2 + d_{2q_3}\phi_3 = c_1\phi_{1q_2} + c_2\phi_{2q_2} + c_3\phi_{3q_2};
\end{aligned} \tag{9b}$$

$$
\begin{aligned}
&a_{3q_1q_1} = b_{3q_2q_2} = c_{3q_3q_3} = 0,\\
&d_{3q_1q_1} = d_{3q_2q_2} = d_{3q_3q_3} = d_{3q_1q_2}\\
&\quad = d_{3q_1q_3} = d_{3q_2q_3} = 0,\\
&2a_{3q_1q_2} + b_{3q_1q_1} = 0,\\
&2a_{3q_1q_3} + c_{3q_1q_1} = 0,\\
&2b_{3q_1q_2} + a_{3q_2q_2} = 0,\\
&2b_{3q_2q_3} + c_{3q_2q_2} = 0,\\
&2c_{3q_1q_3} + a_{3q_3q_3} = 0,\\
&2c_{3q_2q_3} + b_{3q_3q_3} = 0,\\
&a_{3q_2q_3} + b_{3q_1q_3} + c_{3q_1q_2} = 0,\\
&(a_3\phi_1 + b_3\phi_2 + c_3\phi_3)_{q_1} + 2a_{3q_1}\phi_1\\
&\quad + (a_{3q_2} + b_{3q_1})\phi_2 + (a_{3q_3} + c_{3q_1})\phi_3\\
&= a_1\phi_{1q_3} + a_2\phi_{2q_3} + a_3\phi_{3q_3},\\
&(a_3\phi_1 + b_3\phi_2 + c_3\phi_3)_{q_2} + 2b_{3q_2}\phi_2\\
&\quad + (a_{3q_2} + b_{3q_1})\phi_1 + (b_{3q_3} + c_{3q_2})\phi_3\\
&= b_1\phi_{1q_3} + b_2\phi_{2q_3} + b_3\phi_{3q_3},\\
&(a_3\phi_1 + b_3\phi_2 + c_3\phi_3)_{q_3} + 2c_{3q_3}\phi_3\\
&\quad + (a_{3q_3} + c_{3q_1})\phi_1 + (b_{3q_3} + c_{3q_2})\phi_2\\
&= c_1\phi_{1q_3} + c_2\phi_{2q_3} + c_3\phi_{3q_3},\\
&d_{3q_1}\phi_1 + d_{3q_2}\phi_2 + d_{3q_3}\phi_3\\
&= c_1\phi_{1q_3} + c_2\phi_{2q_3} + c_3\phi_{3q_3}.
\end{aligned}
\tag{9c}
$$

先通过观察(9)式并联合(1)式,可进一步假设 $a_i,b_i,c_i,d_i(i=1,2,3)$ 是常数或是关于 q_i,q_j,q_k 组合成的基本函数,将 $a_i,b_i,c_i,d_i(i=1,2,3)$ 的拟解代入(9)式就可解得若干组 $a_i,b_i,c_i,d_i(i=1,2,3)$ 的特殊解,将 $a_i,b_i,c_i,d_i(i=1,2,3)$ 代入(8)式就可解得若干组 U,V,W 的特殊解. 然后分别将解得的 U,V,W 代入(6d),(6e),(6f)式便可解得 K,M,N,则守恒量的一般表达式为

$$
\begin{aligned}
I = \int &-(U\phi_1 + V\phi_2 + W\phi_3 + K\dot{q}_1 + M\dot{q}_2 + N\dot{q}_3)\mathrm{d}t\\
&+ K\mathrm{d}q_1 + M\mathrm{d}q_2 + N\mathrm{d}q_3 + U\mathrm{d}\dot{q}_1 + V\mathrm{d}\dot{q}_2 + W\mathrm{d}\dot{q}_3.
\end{aligned}
\tag{10}
$$

将求得的每组积分乘子 U,V,W,K,M,N 分别代入(10)式,并设法将 $-(U\phi_1+V\phi_2+W\phi_3+K\dot{q}_1+M\dot{q}_2+N\dot{q}_3)\mathrm{d}t + K\mathrm{d}q_1 + M\mathrm{d}q_2 + N\mathrm{d}q_3 + U\mathrm{d}\dot{q}_1 + V\mathrm{d}\dot{q}_2 + W\mathrm{d}\dot{q}_3$ 配成全微分 $\mathrm{d}I(t,q_i,\dot{q}_i)(i=1,2,3)$ 的形式,就可直接得到守恒量. 由于 U,V,W,K,M,N 的解可能有多组,因此相应的守恒量也可能有多个,而求守恒量的关键是求得 U,V,W. 下面将以三质点 Toda 晶格问题为例说明上述 P-S 法求守恒量的过程,并得到了能量积分以外的守恒量.

4. 应用举例

三质点 Toda 晶格问题的 Lagrange 函数为[2]

$$
\begin{aligned}
L = \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2) - \exp(q_1 - q_2)\\
- \exp(q_2 - q_3) - \exp(q_3 - q_1).
\end{aligned}
\tag{11}
$$

由 Lagrange 方程可得系统运动运动微分方程为

$$
\begin{aligned}
\ddot{q}_1 &= \frac{\partial L}{\partial q_1}\\
&= -\exp(q_1 - q_2) + \exp(q_3 - q_1)\\
&= \phi_1(q_1,q_2,q_3),
\end{aligned}
\tag{12a}
$$

$$
\begin{aligned}
\ddot{q}_2 &= \frac{\partial L}{\partial q_2}\\
&= \exp(q_1 - q_2) - \exp(q_2 - q_3)\\
&= \phi_2(q_1,q_2,q_3),
\end{aligned}
\tag{12b}
$$

$$
\begin{aligned}
\ddot{q}_3 &= \frac{\partial L}{\partial q_3}\\
&= \exp(q_2 - q_3) - \exp(q_3 - q_1)\\
&= \phi_3(q_1,q_2,q_3).
\end{aligned}
\tag{12c}
$$

将(12a),(12b),(12c)式代入(7a),(7b),(7c)式得

$$
\begin{aligned}
\ddot{U} = [-\exp(q_1 - q_2) - \exp(q_3 - q_1)]U\\
+ \exp(q_1 - q_2)V + \exp(q_3 - q_1)W,
\end{aligned}
\tag{13a}
$$

$$
\begin{aligned}
\ddot{V} = \exp(q_1 - q_2)U + [-\exp(q_1 - q_2)\\
- \exp(q_2 - q_3)]V + \exp(q_2 - q_3)W,
\end{aligned}
\tag{13b}
$$

$$
\begin{aligned}
\ddot{W} = \exp(q_3 - q_1)U + \exp(q_2 - q_3)V\\
+ [-\exp(q_2 - q_3) - \exp(q_3 - q_1)]W.
\end{aligned}
\tag{13c}
$$

将(13a),(13b),(13c)式求和后可得以下简单关系式:

$$
\ddot{U} + \ddot{V} + \ddot{W} = 0.
\tag{14}
$$

由于系统的 Lagrange 函数不显含时间 t,且 ϕ_1,ϕ_2,ϕ_3 只是关于 $\exp(q_1-q_2),\exp(q_2-q_3),\exp(q_3-q_1)$ 的代数和,故可假设 U,V,W 的拟解为

$$
\begin{aligned}
U = a_1(q_1,q_2,q_3)\dot{q}_1 + b_1(q_1,q_2,q_3)\dot{q}_2\\
+ c_1(q_1,q_2,q_3)\dot{q}_3 + d_1(q_1,q_2,q_3),
\end{aligned}
\tag{15a}
$$

$$
\begin{aligned}
V = a_2(q_1,q_2,q_3)\dot{q}_1 + b_2(q_1,q_2,q_3)\dot{q}_2\\
+ c_2(q_1,q_2,q_3)\dot{q}_3 + d_2(q_1,q_2,q_3),
\end{aligned}
\tag{15b}
$$

$$W = a_3(q_1,q_2,q_3)\dot{q}_1 + b_3(q_1,q_2,q_3)\dot{q}_2 + c_3(q_1,q_2,q_3)\dot{q}_3 + d_3(q_1,q_2,q_3). \quad (15c)$$

系数 $a_i, b_i, c_i, d_i (i = 1,2,3)$ 可以是常数或是 q_i, q_j, q_k 的组合项.将(15)式代入(13a),(13b),(13c)式并考虑(14)式,可解得两组特殊解

$$U_1 = \dot{q}_1, \quad V_1 = \dot{q}_2, \quad W_1 = \dot{q}_3; \quad (16a)$$

$$U_2 = 1, \quad V_2 = 1, \quad W_2 = 1. \quad (16b)$$

将(16)式分别代入(6d),(6e),(6f)式得

$$\begin{aligned} K_1 &= \exp(q_1 - q_2) - \exp(q_3 - q_1), \\ M_1 &= -\exp(q_1 - q_2) + \exp(q_2 - q_3), \quad (17a) \\ N_1 &= -\exp(q_2 - q_3) + \exp(q_3 - q_1); \end{aligned}$$

$$K_2 = 0, \quad M_2 = 0, \quad N_2 = 0. \quad (17b)$$

这两组积分乘子也满足约束方程(6g)—(6k).将(16),(17)式分别代入(10)式得两守恒量

$$\begin{aligned} I_1 &= \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2) \\ &\quad + \exp(q_1 - q_2) \\ &\quad + \exp(q_2 - q_3) + \exp(q_3 - q_1), \quad (18a) \end{aligned}$$

$$I_2 = \dot{q}_1 + \dot{q}_2 + \dot{q}_3. \quad (18b)$$

很明显, I_1 代表系统的能量, I_2 代表系统的总动量.

5. 结　论

本文把扩展 P-S 法用于求三自由度二阶非线性耦合动力学系统的守恒量,得到了 6 个积分乘子所满足的 6 个确定方程和 15 个约束方程,写出了守恒量的一般形式,并讨论了确定积分乘子的方法.最后用扩展 P-S 法求得了三质点 Tada 晶格问题的两个守恒量.P-S 法的理论简洁直观,易于掌握,便于推广到高阶高维系统.随着维数和阶数的增加,积分乘子的计算量大增,人工计算已不可能,必须借助专用软件计算.

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Finding conserved quantities of three-dimensional second-order nonlinear coupled dynamics systems by the extended Prelle-Singer method

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Abstract

In this paper, the extended Prelle-Singer (P-S) method is employed to finding the conserved quantities of three-dimensional second-order nonlinear coupled dynamic systems, the determining equations, the constraint equations of integral factors and the general expression of conserved quantities are obtained. The calculation method of integral factors is disscussed. Finally, two conserved quantities of three-particles Tada crystal lattice problem are found by extended P-S method.

Keywords: extended Prelle-Singer method, three-dimensional nonlinear coupled dynamics systems, conserved quantity

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