EFFECT OF SPACE CHARGE ON THE FREQUENCY OF OSCILLATION IN POSITIVE-GRID OSCILLATORS

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(Received 16 February, 1946)

ABSTRACT

In estimating the frequency of oscillation of a positive-grid oscillator, it is usually assumed that the space charge effect is negligible. Under this assumption, the potential distribution in the space between the plane-parallel electrodes is linear. Thus the electric field in the space is uniform and the average velocity of electron flight can be used to evaluate the frequency of oscillation. The presence of space charge affects the potential distribution in space so that the electric field in this region is no more constant. Hence more exact formula must be used to compute the frequency of oscillation. The calculation is further complicated by the fact that the anode potential is usually made slightly negative with respect to the cathode. In this case, the location of the virtual cathode at which the electrons turn back does not coincide with that if a straight line potential distribution were assumed. The predicated frequency of oscillation is about 10-20 percent lower than that if space charge were neglected.

In the "electron dance" type of oscillators, the frequency of oscillation is primarily determined by the time required for an electron to make a complete excursion between the electrodes under the influence of the impressed fields. For a positive-grid oscillator of the Barkhausen-Kurz type, the usual process in estimating the frequency of oscillation is to assume a certain potential distribution between the electrodes. Then from the equation of motion of electrons in conservative field, the velocity of the electron flight is derived in terms of the potential distribution. Hence for an assigned tube dimension, the time required for the excursion or the frequency of oscillation may be calculated. To ensure simplicity, space charge effects are neglected. For such a case, the potential distribution between the plane-parallel electrodes is a straight line. Because the electric field is uniform, the force on the electrons is constant and the motion that results is one of uniform acceleration. Assume zero initial velocity of the emitted electrons, the average velocity $\bar{v}$ ($=v/2$, where v is the final velocity of electrons when arriving at the electrode having a potential V) may be used to evaluate the time of excursion. Thus, if V_1 is the positive potential applied on the grid of a

Barkhausen-Kurz tube, $-V_2$ be that on its anode, d_1 and d_2 be the cathode to grid and grid to anode separations, respectively, it can be shown that the frequency of oscillation is given by

$$f = \frac{1}{4} \sqrt{\frac{2e}{m}} \frac{V_1^{\frac{1}{2}} (V_1 + V_2)}{d_1(V + V_2) + d_2 V_1} \quad (1)$$

Thus for $d_1 = d_2 = d$,

$$f = \frac{1}{4} \sqrt{\frac{2e}{m}} \frac{V_1^{\frac{1}{2}}}{d} \frac{1 + V_2/V_1}{2 + V_2/V_1} \quad (2)$$

Experimental results indicate that the theoretical frequency is at least 20-30 per cent too low. Barkhausen and Kurz¹ suggested that this large discrepancy is due to the effect of the space charge which distorts the potential distribution such that the path of electron flight is shortened, and the frequency of oscillation is thereby raised. Hollmann² predicated that the space charge could affect the frequency of oscillation in the other direction due to change of electron velocity near the emitter. But no quantitative data were given. In this paper, the effect of space charge on the frequency of oscillation is investigated analytically. The results indicate that the frequency is actually lowered (see discussion).

The potential distribution in plane-parallel positive-grid tube.

The effect of space charge on the potential distribution between plane-parallel electrodes has been studied extensively by Haeff³ and Fay and others.⁴ Modifying their results to fit the present problem, it can be shown that the potential distribution between the cathode to grid and grid to anode spaces are as shown in Fig. 1. In the first region, that is, in the region between the cathode and the positive grid, the potential distribution satisfies Child's equation. In the second region, that is, in the region between the positive grid and the anode, it cuts the zero axis at d_0 and joins with $-V_2$ by a straight line whose slope is the same as that of the potential curve at d_0 . Express it mathematically, we have, in the first region, the potential distribution is given by Child's equation

$$V = K^{\frac{2}{3}} I_0^{\frac{2}{3}} x^{\frac{4}{3}}, \quad (3)$$

1. Barkhausen, H and Kurz, K. *Physik. Zeit.* Bd. 21, (1920).
2. Hollmann, H. E. *Physik und Technik der Ultrakurzen Wellen*, Bd. I, p. 142.
3. Salzberg, B. and A. V. Haeff, *R. C. A. Review*, 2, (1938), 336.
4. Fay, C. E., A. L. Samuel and W. Shockley, *Bell System Tech. Jour.* 17, (1938), 49-79.

where V is the space potential, $K = \frac{9\pi\sqrt{m}}{2e}$, I_0 is the current density in that region and x is the distance from cathode to grid. In the second region, space charge exists only between grid and d_0 , then

$$\frac{x^1}{d} = \frac{1}{P} \left[\left(1 - 2\sqrt{\frac{M_1}{V_1}}\right) \sqrt{1 + \sqrt{\frac{M_1}{V_1}}} - \left(\sqrt{\frac{V}{V_1}} - 2\sqrt{\frac{M_1}{V_1}}\right) \sqrt{\sqrt{\frac{V}{V_1}} + \sqrt{\frac{M_1}{V_1}}} \right], \quad (4)$$

where $P = \frac{4/3 KI_0^{\frac{1}{2}} d}{V_1^{\frac{3}{4}}}$, V_1 is the potential on grid and M_1 is a function of the slope of the potential curve at the zero cross (at d_0) and x' is the new distance counted from grid toward the anode. Between the potential zero at d_0 and the anode, there is no space charge, the potential distribution is a straight line whose slope is given by

$$\left(\frac{M_1}{V_1}\right)^{\frac{1}{2}} = \frac{1}{P^2} \frac{1}{(1 - \frac{d_0}{d})^2} \left(\frac{V_2}{V_1}\right)^2, \quad (5)$$

$$\text{or} \quad \frac{d_0}{d} = 1 - \frac{1}{P} \left(\frac{V_2}{V_1}\right) \frac{1}{\left(\frac{M_1}{V_1}\right)^{\frac{1}{4}}}. \quad (6)$$

Eqs. (4), (5) or (6) are interdependent and the evaluation of them is very involved. Let us use another equation which is obtained from Eq. (4) by letting $x' = d_0$ and $V = 0$,

$$\text{or} \quad R = \frac{d_0}{d} P = \left(1 - 2\sqrt{\frac{M_1}{V_1}}\right) \sqrt{1 + \sqrt{\frac{M_1}{V_1}}} + 2 \left(\frac{M_1}{V_1}\right)^{\frac{3}{4}}, \quad (7)$$

and rewrite Eq. (6) as

$$P = R + \left(\frac{V_2}{V_1}\right) \frac{1}{\left(\frac{M_1}{V_1}\right)^{\frac{1}{4}}}. \quad (8)$$

Then from Eq. (7), let us calculate R for assigned values of M_1/V_1 . And from Eq. (8), the corresponding values of P for different values of V_2/V_1 are evaluated. Using the plot of P against V_2/V_1 for different constant values of M_1/V_1 , the values of M_1/V_1 for different values of V_2/V_1 at several constant values of P can be evaluated. And thus Eq. (4) is solved.

The frequency of oscillation in a positive-grid oscillator

If the potential distribution is not linear, the force on electrons is no more constant, and the average velocity should not be used to calculate the time of flight between the electrodes. It can best be evaluated from the relation

$$t = \sqrt{\frac{m}{2e}} \int \frac{dx}{V^{\frac{1}{2}}}, \quad (9)$$

provided the potential distribution between the electrodes is known. Utilizing Eq. (3) and Eq. (9), the time of flight between cathode and grid is

$$t_1 = 3\sqrt{\frac{m}{2e}} \frac{d_1}{\sqrt{V_1}}, \quad (10)$$

where d_1 is the cathode to grid distance. The time of flight between the grid and anode can be calculated from Eqs. (4) and (9) and is

$$t_2 = 3\sqrt{\frac{m}{2e}} \frac{d_2}{\sqrt{V_1}} \frac{\sqrt{1 + \sqrt{M_1/V_1}} - \sqrt[4]{M_1/V_1}}{P}. \quad (11)$$

The time required for a complete electron excursion is, for $d_1 = d_2 = d$,

$$t = 2(t_1 + t_2) = 6\sqrt{\frac{m}{2e}} \frac{d}{\sqrt{V_1}} \left[1 + \frac{\sqrt{1 + \sqrt{M_1/V_1}} - \sqrt[4]{M_1/V_1}}{P} \right], \quad (12)$$

or the frequency of oscillation is

$$f = \frac{1}{t} = \frac{1}{6} \sqrt{\frac{2e}{m}} \frac{\sqrt{V_1}}{d} \frac{1}{1 + \frac{\sqrt{1 + \sqrt{M_1/V_1}} - \sqrt[4]{M_1/V_1}}{P}}. \quad (13)$$

The calculated frequency of oscillation is plotted on Fig. 2.

Discussions

1. It is seen from Eqs. (3) and (4) or the plot on Fig. 1, that the effect of the space charge is to distort the potential distribution in such a way that the acceleration of the electrons between the electrodes is not uniform. The transit time thus calculated from Eq. (12) indicates that electrons will take a longer time to make a complete excursion between the electrodes than if straight-line potential distribution were assumed. This means the frequency of oscillation is lowered.

2. Comparing Eqs. (2) and (13), it is noticed that the fundamental Barkhausen-Kurz relation is not violated, that is, the frequency of oscillation is proportional to $\sqrt{V_1}/d$. However, the constants of proportionality have been changed in two ways. In the first place, the numerical constant has been changed from $1/4$ to $1/6$; and in the second place, the constant that varies with the ratio— V_2/V_1 has been changed to a more complicated function including both the effects of V_2/V_1 and P (which involves the variation of the space current density I_0).

3. The discrepancy between the numerical constants $1/4$ and $1/6$ comes from the fact that in one case, the potential distribution is proportional to the distance, while in the other case, it is proportional to $x^{4/3}$. It is almost impossible in trying to deduce one from the other. This is because in case we have space charge, then by Child's law, the charge density is proportional to $1/x^{2/3}$. That is to say, in the immediate vicinity of the cathode, the charge density is infinite no matter how small the total space current is while in the space-charge-free condition, it is required to have zero charge density everywhere. This discrepancy brings about a frequency difference of about thirty percent.

4. From Eq. (11) and the plot on Fig. 1, it is seen that from cathode to grid, if the space charge is varied by varying the filament current, the transit time is not changed at all. In actual case, with increased filament current, the initial velocity may come into picture. This effect distorts the potential distribution such that the latter will cut the zero axis in front of the cathode and will have a finite slope. A potential zero is formed where the excursive electrons make their turns. Thus the distance travelled by the electrons is reduced and their velocity is increased. The frequency of oscillation is therefore increased. Experiments by Barkhausen and Kurz¹ supported this interpretation.

5. The effect of varying V_2 is shown in Fig. 2. As V_2 increases in the negative direction, the frequency of oscillation is increased. This is because the location d_0 where the potential curve cuts the zero axis shifts toward the grid for larger negative values of V_2/V_1 . This cuts down the distance travelled by the electrons and causes the frequency of oscillation to increase.

6. The location of the potential zero for a space-charge-free condition does not coincide with that if space charge were present, as is seen from Fig. 1 and Eq. (6). It is clear that d_0 is different for different values of P 's. Since M_1/V_1 is

proportional to $1/P^2$, hence, as P increases, the location of the potential zero decreases as it should be. Thus the frequency of oscillation is increased. However, it can be seen from Fig. 2, that the frequency variation due to changes in P is not very pronounced.

7. Although the time required for the electrons to make a complete excursion between the electrodes is regarded as the reciprocal of the frequency of oscillation in this type of oscillators, it seems to the author that one half of this time is a more appropriate value for the frequency. Since from pure energy relation, it is considered that only those electrons starting from the cathode at the time when the grid potential is decreasing and cross the grid plane at the time when the grid potential changes its sign may actually give up their energy to the oscillating system. It is therefore conceivable that the half period of the excursion should be used to evaluate the frequency of oscillation. If this is done, the experimental and theoretical (with the effect of space charge) frequencies of oscillation could have a much better check. It is therefore suggested that the frequency of oscillation is double that given by Eq. (13).

Acknowledgment

The writer wishes to thank Dr. D. Y. Mas for his valuable discussions in the course of this investigation.

Fig. 1. Potential distributions in a plane-parallel positive-grid tube, showing the effect of the space charge.

Fig. 2. Frequency of oscillation as calculated by Eqs. (2) and (13).