物理学报 Acta Physica Sinica
Chinese Physical Society
Institute of Physics, CAS

Tavis-Cummings模型中的几何量子失协特性

程景　单传家　刘继兵　黄燕霞　刘堂昆

Geometric quantum discord in Tavis-Cummings model

Cheng Jing　Shan Chuan-Jia　Liu Ji-Bing　Huang Yan-Xia　Liu Tang-Kun

引用信息 Citation: Acta Physica Sinica, **67**, 110301 (2018)　DOI: 10.7498/aps.67.20172699
在线阅读 View online: http://dx.doi.org/10.7498/aps.67.20172699
当期内容 View table of contents: http://wulixb.iphy.ac.cn/CN/Y2018/V67/I11

物理学报 Acta Phys. Sin. Vol. 67, No. 11 (2018) 110301

Tavis-Cummings模型中的几何量子失协特性*

程景 单传家† 刘继兵 黄燕霞 刘堂昆

(湖北师范大学物理与电子科学学院, 黄石 435002)

(2017 年 12 月 20 日收到; 2018 年 3 月 14 日收到修改稿)

采用几何量子失协的计算方法, 通过改变两原子初始状态、腔内光子数和偶极-偶极相互作用强度, 研究了 Tavis-Cummings 模型中的几何量子失协特性. 结果表明: 几何量子失协都是随时间周期性振荡的, 选取适当的初态可以使两原子一直保持失协状态, 增加腔内光子数和偶极相互作用对几何量子失协有积极的影响.

关键词: 量子纠缠, 几何量子失协, 偶极-偶极相互作用

PACS: 03.65.Ud, 42.50.Pq **DOI:** 10.7498/aps.67.20172699

1 引 言

量子纠缠是量子力学最基本的概念, 是一种量子关联, 也是量子物理与经典物理最本质的区别, 在量子信息中被广泛地应用, 例如量子密钥分配[1]、量子隐形传态[2]、量子纠错[3]和量子网络等. 目前人们对 Tavis-Cummings (T-C) 模型进行了更广泛的研究, Bogoliubov 等[4]对加入克尔非线性介质的 T-C 模型进行了讨论; 左战春和夏云杰[5]对 T-C 模型中三体纠缠态纠缠量进行了讨论; 郭亮和梁先庭[6]研究了 T-C 模型中光场和原子以及原子与原子之间的纠缠演化; 张国锋和卜晶晶[7]研究了共振和非共振情况下非简并双光子 T-C 模型中原子与原子之间的纠缠演化. 但是, 实验上和理论上证实了量子纠缠不包含所有的量子关联. 为更加全面地描述量子关联, Ollivier 和 Zurek[8]提出了量子失协 (quantum discord, QD); 胡要花等[9]研究了强度相关耦合双 Jaynes-Cummings (J-C) 模型中的纠缠和 QD; 贺志和李龙武[10]研究了两能级原子在共同环境下的量子关联动力学; Ali 等[11]发现纠缠与 QD 并没有直接的大小关系. 量子纠缠不完全等同于非经典关联, 有的系统即使量子纠缠等于 0, 但是仍然存在量子失协, 所以 QD 引起了人们浓厚的兴趣. 但是仅对于一些特殊的态, 才能得到 QD 的精确解析. 为了克服这个问题, Dakic 等[12]提出了几何量子失协 (geometric measure of quantum discord, GQD) 的方法, 用此方法解析两体问题相对于 QD 要简单很多. 樊开明和张国锋[13]研究了阻尼 J-C 模型中两原子的量子关联动力学. 对 GQD 的研究更为广泛[14−20]. 单传家和夏云杰[21]研究了 T-C 模型中两原子的纠缠特性, 但并未研究该模型中的 GQD 特性.

在本文中, 我们考虑两个原子初始时刻处于纠缠态, 并不忽略原子之间的偶极相互作用, 研究 GQD 的演化规律及其受初始原子态、光场数和偶极-偶极相互作用强度的影响.

2 理论模型

考虑两个两能级原子与单模光场的相互作用, 其原子间的偶极-偶极相互作用也被考虑在内. 此情况下的系统哈密顿量 H 为 (设 $\hbar = 1$)

$$H = \omega \boldsymbol{a}^{+}\boldsymbol{a} + \frac{1}{2}\sum_{i=1}^{2}\omega_i \boldsymbol{\sigma}_i^{z} + \sum_{i=1}^{2} g_i (a^{+}\boldsymbol{\sigma}_i^{-} + a\boldsymbol{\sigma}_i^{+}) + \varOmega(\boldsymbol{\sigma}_1^{+}\boldsymbol{\sigma}_2^{-} + \boldsymbol{\sigma}_2^{+}\boldsymbol{\sigma}_1^{-}), \tag{1}$$

---

* 国家自然科学基金 (批准号: 11404108) 和湖北省自然科学基金 (批准号: 2016CFB639) 资助的课题.

† 通信作者. E-mail: scj1122@163.com

其中ω_1和ω_2为原子的本征跃迁频率, ω为腔场频率; $\boldsymbol{a}$和$\boldsymbol{a}^+$分别是光子的湮灭和产生算符; g为原子与光场间的耦合系数; $\varOmega$为原子间偶极-偶极相互作用强度; $\boldsymbol{\sigma}$, $\boldsymbol{\sigma}_1^{\pm}$, $\boldsymbol{\sigma}_2^{\pm}$为原子的自旋算符.

若$|g\rangle$和$|e\rangle$分别是原子的基态和激发态, 则原子A的赝自旋算符为

$$\begin{aligned}\boldsymbol{\sigma}_1^z &= |e_{\mathrm{A}}\rangle\langle e_{\mathrm{A}}| - |g_{\mathrm{A}}\rangle\langle g_{\mathrm{A}}|,\\ \boldsymbol{\sigma}_1^- &= |g_{\mathrm{A}}\rangle\langle e_{\mathrm{A}}|,\\ \boldsymbol{\sigma}_1^+ &= |e_{\mathrm{A}}\rangle\langle g_{\mathrm{A}}|,\end{aligned}\tag{2}$$

原子B的赝自旋算符为

$$\begin{aligned}\boldsymbol{\sigma}_2^z &= |e_{\mathrm{B}}\rangle\langle e_{\mathrm{B}}| - |g_{\mathrm{B}}\rangle\langle g_{\mathrm{B}}|,\\ \boldsymbol{\sigma}_2^- &= |g_{\mathrm{B}}\rangle\langle e_{\mathrm{B}}|,\\ \boldsymbol{\sigma}_2^+ &= |e_{\mathrm{B}}\rangle\langle g_{\mathrm{B}}|.\end{aligned}\tag{3}$$

在相互作用绘景中, 考虑共振条件($\omega = \omega_1 = \omega_2$), (1)式可变为

$$\begin{aligned}H_{\mathrm{I}} = {}& \sum_{i=1}^{2} g_i(a^+\boldsymbol{\sigma}_i^- + a\boldsymbol{\sigma}_i^+)\\ & + \varOmega(\boldsymbol{\sigma}_1^+\boldsymbol{\sigma}_2^- + \boldsymbol{\sigma}_2^+\boldsymbol{\sigma}_1^-).\end{aligned}\tag{4}$$

考虑共振情况且$g_1 = g_2$, 假设初始两原子处于纠缠态, 光场处于粒子数态, 则系统初始状态为

$$\psi(0) = \cos\theta\,|e_{\mathrm{A}}, g_{\mathrm{B}}, n\rangle + \sin\theta\,|g_{\mathrm{A}}, e_{\mathrm{B}}, n\rangle,\tag{5}$$

所以在任何时刻的态矢可表示为

$$\begin{aligned}\psi(t) = {}& C_1(t)\,|e_{\mathrm{A}}, e_{\mathrm{B}}, n-1\rangle + C_2(t)\,|e_{\mathrm{A}}, g_{\mathrm{B}}, n\rangle\\ & + C_3(t)\,|g_{\mathrm{A}}, e_{\mathrm{B}}, n\rangle\\ & + C_4(t)\,|g_{\mathrm{A}}, g_{\mathrm{B}}, n+1\rangle.\end{aligned}\tag{6}$$

将(4), (5)和(6)式代入相互作用绘景的薛定谔方程

$$\mathrm{i}\frac{\partial\,|\psi(t)\rangle}{\partial t} = H_{\mathrm{I}}\,|\psi(t)\rangle,\tag{7}$$

通过计算可以得到

$$\begin{cases}C_1(t) = \dfrac{(\cos\theta+\sin\theta)g\sqrt{n}(\,\mathrm{e}^{\mathrm{i}at} - \mathrm{e}^{\mathrm{i}bt})}{\varDelta},\\[2ex] C_2(t) = \dfrac{-a(\cos\theta+\sin\theta)\,\mathrm{e}^{\mathrm{i}at} + b(\cos\theta+\sin\theta)\,\mathrm{e}^{\mathrm{i}bt}}{2\varDelta} + \dfrac{\cos\theta-\sin\theta}{2}\,\mathrm{e}^{\mathrm{i}\varOmega t},\\[2ex] C_3(t) = \dfrac{-a(\cos\theta+\sin\theta)\,\mathrm{e}^{\mathrm{i}at} + b(\cos\theta+\sin\theta)\,\mathrm{e}^{\mathrm{i}bt}}{2\varDelta} + \dfrac{\sin\theta-\cos\theta}{2}\,\mathrm{e}^{\mathrm{i}\varOmega t},\\[2ex] C_4(t) = \dfrac{(\cos\theta+\sin\theta)g\sqrt{n+1}(\,\mathrm{e}^{\mathrm{i}at} - \mathrm{e}^{\mathrm{i}bt})}{\varDelta},\end{cases}\tag{8}$$

式中$a = (-\varOmega - \varDelta)/2$, $b = (-\varOmega + \varDelta)/2$, $\varDelta = \sqrt{8g^2(1+2n) + \varOmega^2}$.

(6)式对光场求迹, 在原子基$|ee\rangle$, $|eg\rangle$, $|ge\rangle$, $|gg\rangle$下, 约化密度矩阵$\boldsymbol{\rho}_{\mathrm{AB}}(t)$为

$$\boldsymbol{\rho}_{\mathrm{AB}}(t) = \begin{pmatrix}|C_1|^2 & 0 & 0 & 0\\ 0 & |C_2|^2 & C_2C_3^* & 0\\ 0 & C_3C_2^* & |C_3|^2 & 0\\ 0 & 0 & 0 & |C_4|^2\end{pmatrix}.\tag{9}$$

3 数值计算和讨论

Dakic等[12]提出了GQD, 任何一个2 bit的量子态$\boldsymbol{\rho}^{\mathrm{ab}}$都有如(10)式所示的Bloch表示:

$$\begin{aligned}\boldsymbol{\rho}^{\mathrm{ab}} = \frac{1}{4}\Big(&\boldsymbol{I}^{\mathrm{a}}\otimes\boldsymbol{I}^{\mathrm{b}} + \sum_{i=1}^{3}\boldsymbol{x}_i\boldsymbol{\sigma}_i\otimes I^{\mathrm{b}} + \sum_{i=1}^{3}\boldsymbol{y}_i\boldsymbol{I}^{\mathrm{a}}\otimes\boldsymbol{\sigma}_i\\ & + \sum_{i,j=1}^{3} t_{ij}\boldsymbol{\sigma}_i\otimes\boldsymbol{\sigma}_j\Big),\end{aligned}\tag{10}$$

式中$\boldsymbol{x}_i = \mathrm{tr}\rho(\boldsymbol{\sigma}_i\otimes\boldsymbol{I}^{\mathrm{b}})$, $\boldsymbol{y}_i = \mathrm{tr}\,\rho(\boldsymbol{I}^{\mathrm{a}}\otimes\boldsymbol{\sigma}_i)$, $t_{ij} = \mathrm{tr}\,\rho(\boldsymbol{\sigma}_i\otimes\boldsymbol{\sigma}_j)$, $\boldsymbol{x} = (x_1, x_2, x_3)^{\mathrm{T}}$, $\boldsymbol{y} = (y_1, y_2, y_3)^{\mathrm{T}}$是边缘态$\boldsymbol{\rho}^{\mathrm{a}}$和$\boldsymbol{\rho}^{\mathrm{b}}$的Bloch矢量, 而$\boldsymbol{T} = \{t_{ij}\}$是两体量子态的关联矩阵. $\boldsymbol{\rho}_{\mathrm{AB}}$的GQD为

$$D_{\mathrm{G}}(\boldsymbol{\rho}^{\mathrm{ab}}) = \frac{1}{4}(\|x\|^2 + \|T\|^2 - K_{\max}),\tag{11}$$

式中$K_{\max}$是矩阵$\boldsymbol{K} = \boldsymbol{x}\boldsymbol{x}^{\mathrm{T}} + \boldsymbol{T}\boldsymbol{T}^{\mathrm{T}}$的最大本征值, 上标T表示矩阵转置. 利用(11)式进行数值计算, 通过图1—图3展示GQD的演化规律.

图1描述的是改变两原子的初始状态时, GQD的演化规律. 当$\theta = \pi/4$, $\pi/2$时, GQD随时间周期性振荡, 但可以发现当初态为最大纠缠态时, 初始的GQD最大并随着时间减小到零后再回复至最大值; 而初态为分离态时, GQD从零开始增大, 减小到某个值时再增大后继续减小到零, 但不能达到最大值. 当$\theta = 3\pi/8$, $5\pi/8$时, 初始时刻的GQD是一样的, 但随着时间推移, 图1(b)中曲线的GQD先减小到零之后恢复到初始值, 而图1(d)中曲线一直处于失协状态并且GQD的值不会低于初始值.

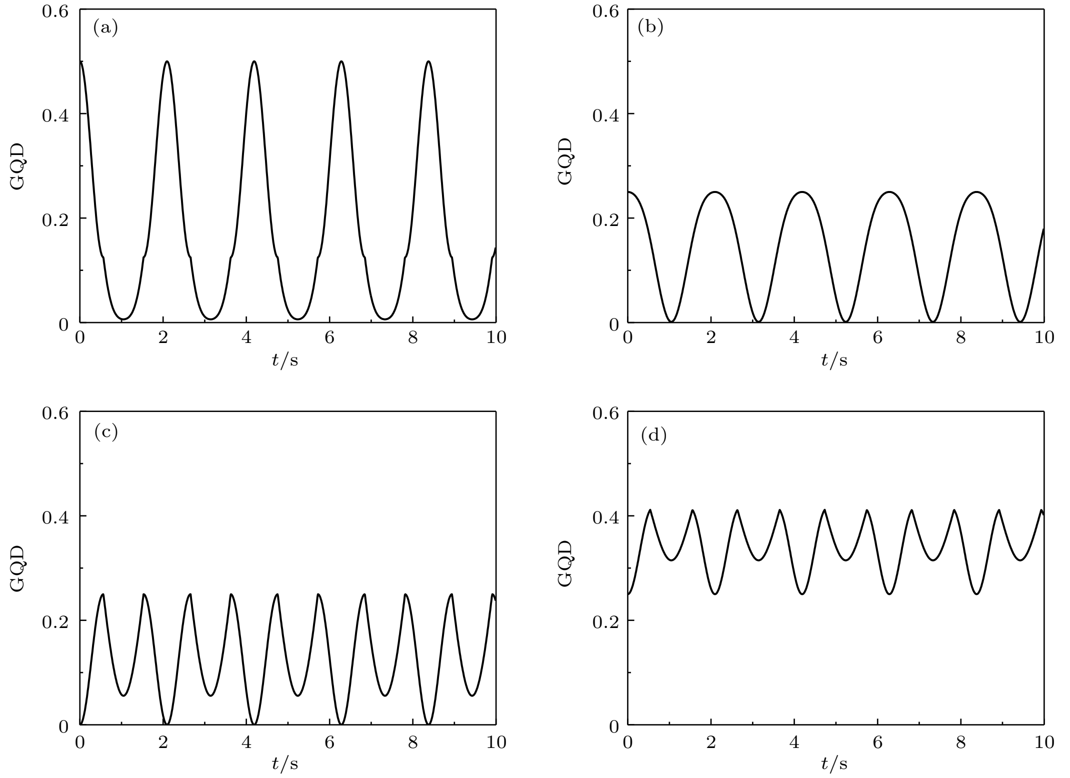

图1 当 $g=1$, $\Omega=1$, $n=0$ 时 GQD 随时间 t 的演化 (a) $\theta=\pi/4$; (b) $\theta=3\pi/8$; (c) $\theta=\pi/2$; (d) $\theta=5\pi/8$

Fig. 1. Geometric quantum discord of two atoms as a function of the time for different initial state (parameters: $g=1$, $\Omega=1$, $n=0$): (a) $\theta=\pi/4$; (b) $\theta=3\pi/8$; (c) $\theta=\pi/2$; (d) $\theta=5\pi/8$.

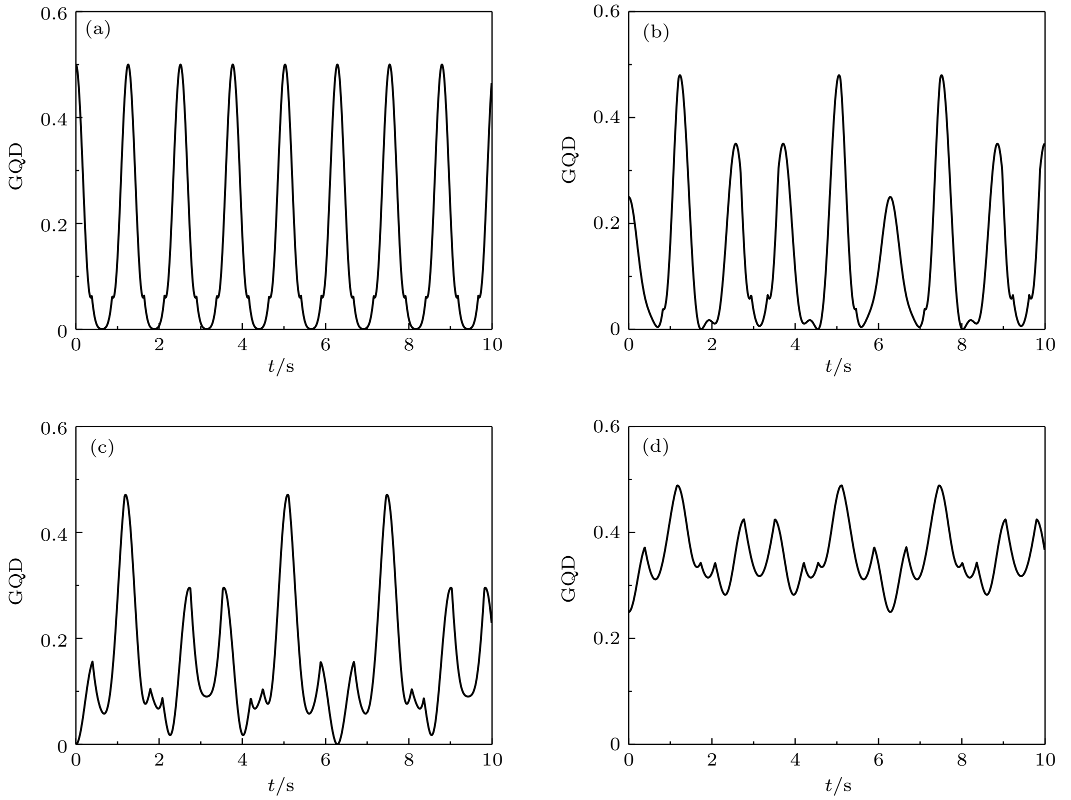

图2 当 $g=1$, $\Omega=1$, $n=1$ 时 GQD 随时间 t 的演化 (a) $\theta=\pi/4$; (b) $\theta=3\pi/8$; (c) $\theta=\pi/2$; (d) $\theta=5\pi/8$

Fig. 2. Geometric quantum discord of two atoms as a function of the time for different initial state (parameters: $g=1$, $\Omega=1$, $n=1$): (a) $\theta=\pi/4$; (b) $\theta=3\pi/8$; (c) $\theta=\pi/2$; (d) $\theta=5\pi/8$.

图2描述的是改变腔场中的光子数, GQD的演化规律. 从图中我们可以看到改变腔场的光子数量子GQD也是周期性演化. 通过图1与图2的对比发现增加光子数后, 使振荡周期变大并且长时间处于失协状态[如图1(c)和图2(c)所示], 也会增强GQD强度, 使失协强度更接近0.5[如图1(b)和图2(b)所示]. 当$\theta = \pi/4$时, 增加腔内光子数却加快了失协振荡频率[如图1(a)和图2(a)所示].

图3描述的是改变原子间偶极-偶极相互作用强度, 腔中两原子的GQD的演化规律. 对比图3与图2可以发现, 增强偶极-偶极相互作用强度, GQD也是周期性振荡并使两原子一直处于失协状态[如图3(a)和图2(a)所示]. 当初态不为最大纠缠态时, 增大偶极-偶极相互作用的强度, 可以增大GQD并且可以增大振荡周期[如图3(b)和图2(b), 图3(d)和图1(d)所示]. 并且发现一个有趣的现象是, 当$\theta = 3\pi/8$时, 增大偶极-偶极相互作用强度, 进入腔后, GQD的值先增大, 再减小, 这与增加腔内光子数的情况有所不同.

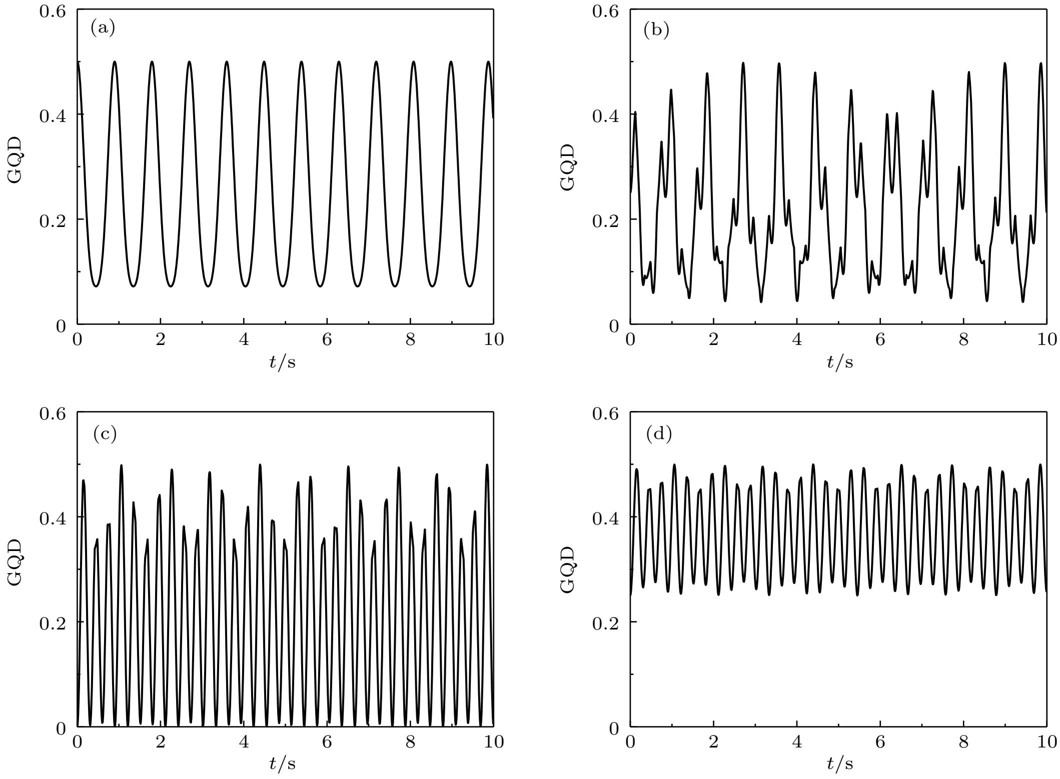

图3 当$g = 1$, $\varOmega = 5$时GQD随时间t的演化规律 (a) $n = 1$, $\theta = \pi/4$; (b) $n = 1$, $\theta = 3\pi/8$; (c) $n = 0$, $\theta = \pi/2$; (d) $n = 0$, $\theta = 5\pi/8$

Fig. 3. Geometric quantum discord of two atoms as a function of the time for different initial state and different Fock state of the field (parameters: $g = 1$, $\varOmega = 5$): (a) $n = 1$, $\theta = \pi/4$; (b) $n = 1$, $\theta = 3\pi/8$; (c) $n = 0$, $\theta = \pi/2$; (d) $n = 0$, $\theta = 5\pi/8$.

4 结 论

通过改变两原子初始状态、腔内光子数和偶极-偶极相互作用强度, 研究了T-C模型中的GQD特性. 结果表明: 选取适当的初态可以使两原子一直处于失协状态, 同时发现适当地增强偶极-偶极相互作用强度可以使两原子一直处于失协状态. 增加腔内光子数或增强偶极-偶极相互作用强度都可以使GQD有略微的增大. 本文的研究结果对量子纠缠态制备、QD的操控有一定的指导意义.

参考文献

[1] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661

[2] Bennett C H, Brassard G, Grepean C, Jozsa R, Peres A, Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[3] Shor P W 1992 *Phys. Rev. Lett.* **52** 2493

[4] Bogoliubov N M, Bulloughz R K, Timonenx 1996 *J. Phys. A: Math. Gen.* **19** 6305

[5] Zuo Z C, Xia Y J 2003 *Acta Phys. Sin.* **52** 2687 (in Chinese) [左战春, 夏云杰 2003 物理学报 **52** 2687]

[6] Guo L, Liang X T 2009 *Acta Phys. Sin.* **58** 50 (in Chinese) [郭亮, 梁先庭 2009 物理学报 **58** 50]

[7] Zhang G F, Bu J J 2010 *Acta Phys. Sin.* **59** 1462 (in Chinese) [张国锋, 卜晶晶 2010 物理学报 **59** 1462]

[8] Ollivier H, Zurek W H 2001 *Phys. Rev. Lett.* **88** 017910

[9] Hu Y H, Tan Y G, Liu Q 2013 *Acta Phys. Sin.* **62** 074202 (in Chinese) [胡要花, 谭勇刚, 刘强 2013 物理学报 **62** 074202]

[10] He Z, Li L W 2013 *Acta Phys. Sin.* **18** 180301 (in Chinese) [贺志, 李龙武 2013 物理学报 **18** 180301]

[11] Ali M, Rau A R P, Alber G 2010 *Phys. Rev. A* **81** 359

[12] Dakic B, Vedral V, Brukner C 2010 *Phys. Rev. Lett.* **105** 4649

[13] Fan K M, Zhang G F 2013 *Acta Phys. Sin.* **62** 130301 (in Chinese) [樊开明, 张国锋 2013 物理学报 **62** 130301]

[14] Hu M L, Tian D P 2014 *Ann. Phys.* **343** 132

[15] LI Y J, Liu J M, Zhang Y 2014 *Chin. Phys. B* **23** 110306

[16] Chang L, Luo S 2013 *Phys. Rev. A* **82** 034302

[17] Zhu H J, Zhang G F 2014 *Chin. Phys. B* **23** 120306

[18] Song W, Yu L B, Dong P, Li D C, Yang M, Cao Z L 2013 *Sci. China: Phys. Mech. Astron.* **56** 737

[19] Xiao Y L, Li T, Fei S M, Jing N, Wang Z X 2016 *Chin. Phys. B* **25** 030301

[20] Zou H M, Fang M F 2016 *Chin. Phys. B* **25** 090302

[21] Shan C J, Xia Y J 2006 *Acta Phys. Sin.* **55** 1585 (in Chinese) [单传家, 夏云杰 2006 物理学报 **55** 1585]

物 理 学 报 Acta Phys. Sin. Vol. 67, No. 11 (2018) 110301

Geometric quantum discord in Tavis-Cummings model*

Cheng Jing Shan Chuan-Jia† Liu Ji-Bing Huang Yan-Xia Liu Tang-Kun

(*College of Physics and Electronic Science, Hubei Normal University, Huangshi 435002, China*)

(Received 20 December 2017; revised manuscript received 14 March 2018)

Abstract

Quantum entanglement plays a key role in quantum information and quantum computation and thus attracts much attention in many branches of physics both in theory and in experiment. But recent studies revealed that some separable states (non-entangled state) may speed up certain tasks over their classical counterparts and may also possess certain kinds of quantum correlations. For example, geometric quantum discord, which is a more general quantum correlation measure than entanglement, can be nonzero for some separable states. From a practical point of view, it is proposed that the geometric quantum discord be responsible for the power of many quantum information processing tasks. In order to capture such correlations, Ollivier and Zurek introduced quantum discord, which measures the discrepancy between two natural yet different quantum analogues of two classically equivalent expressions of mutual information. However, the calculation of quantum discord is based on numerical maximization procedure, and there are few analytical expressions even for a two-qubit state. In order to obtain the analytical results of quantum discord, a geometric measure of quantum discord which measures the quantum correlations through the minimum Hilbert-Schmidt distance between the given state and zero discord state is introduced. Geometric quantum discord is defined as an effective measure of quantum correlation, and the geometric quantum discord through the minimal distance between the quantum state and the set of zero-discord states in a bipartite quantum system can be worked out. In this paper, by using the geometric quantum discord measurement method, the geometric quantum discord in Tavis-Cummings model is investigated, and the influences of the initial state purity, entanglement degree, dipole-dipole coupling intensity between two atoms, and field in the Fock state on the evolution characteristic of geometric quantum discord are analyzed. The results show that the geometric quantum discord appears periodically. It initially decreases to a minimum value, and then turns out to be increased for different initial states. The rigorous analysis and numerical results reveal that when we take a suitable initial state, the geometric quantum discord of two atoms can be kept in correlation. When the atoms are in the different initial states, the quantum properties of the system are significant. The photon number of the field can lead the quantum discord to be weakened. Geometric quantum discord can be increased by increasing the cavity photon number and the dipole-dipole coupling intensity. Geometric quantum discord can be enhanced obviously by increasing the strength of the dipole-dipole coupling interaction. The conclusions may conduce to the understanding of quantum correlation for the other systems from the view of geometric quantum discord.

Keywords: quantum entanglement, geometric quantum discord, the dipole-dipole coupling intensity

PACS: 03.65.Ud, 42.50.Pq **DOI:** 10.7498/aps.67.20172699

---

* Project supported by the National Natural Science Foundation of China (Grant No. 11404108) and the Natural Science Foundation of Hubei Province, China (Grant No. 2016CFB639).

† Corresponding author. E-mail: scj1122@163.com